I. Solve the problem.

1. One method to determine the time since an animal died is to estimate the percentage of carbon-14 remaining in its bones. The percent $P$ in decimal form of carbon-14 remaining $x$ years is given by $P(x) = e^{-0.000121x}$. Approximate (to the nearest whole year) the age of a fossil if there is 41% of carbon-14 remaining. (hint $P(x)=0.41$)

2. Suppose the number of Quickie hamburgers (in millions) served yearly from 1987 to 2000 can be modeled by $f(x) = 39.8e^{0.12x}$. In this formula $x=0$ corresponds to 1987 and $x=13$ corresponds to 2000.
   a) Approximate how many million (rounded to the nearest million) Quickie hamburgers will be sold in 2000.
   b) In what year (to the nearest year) did Quickie serve 117.2 million hamburgers?

3. Find the amount of money in an account after 10 years if $4200 is deposited at 6% annual interest compounded semiannually.

4. What will be the amount in an account with initial principal of $9000 if interest is compounded continuously at an annual rate of 3.25% for 7 years?

5. $9000 is invested at 8% compounded quarterly. In how many years will the account have grown to $14,500? Round your answer to the nearest tenth of a year.

6. Noriko invested $10,000 at 7% compounded semiannually. In how many years will Noriko’s investment have quadrupled? Round your answer to the nearest tenth of a year.

7. Under ideal conditions, a population of rabbits has an exponential growth rate of 11.5% per day. Consider an initial population of 900 rabbits.
   a) Find the exponential growth function. (Do not use $e$ in the function)
   b) How many rabbits will you have in 5.5 days? (to the nearest rabbit)
   c) How many days will it take to have 7621 rabbits?

8. A colony of bacteria decreases in size every 6 hours by half of its previous amount when an antibiotic is given. The original amount was estimated at 15000.
   a) Find the exponential decay function. (Do not use $e$ in the function)
   b) How many bacteria are left after 3 hours?
   c) How many hours will it take till the colony is down to 1052 bacteria? (to the nearest hour)
9. Using the data in the table: round all coefficients and r values to three decimal places.

<table>
<thead>
<tr>
<th>Years after 1995</th>
<th>Number of Widgets Produced (in millions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.9</td>
</tr>
<tr>
<td>2</td>
<td>3.0</td>
</tr>
<tr>
<td>3</td>
<td>4.6</td>
</tr>
<tr>
<td>4</td>
<td>6.3</td>
</tr>
<tr>
<td>5</td>
<td>8.6</td>
</tr>
</tbody>
</table>

a) Find an exponential function to model the data and its r value.
b) Find a natural logarithmic function to model the data and its r value.
c) Which model is the better fit and why?
d) Use the best fit model, how many widgets (in millions) are produced in 2002?

II. Solve the equation, algebraically. Round to three decimal places.
10. $e^{0.386x} = 21$
11. $4^{(x-3)} = 15$
12. $e^{\ln x} = 11$
13. $\ln x - \ln (x - 4) = \ln 2$
14. $\log_5(x + 9) + \log_5(x - 9) = 1$

III. Use the change of base formula to approximate to four decimal places.
15. $\log_{5.2} 178$
16. $\log_3 0.435$

IV. Expand the expression without using exponents.
17. $\log_a (7x^3yz^4)$

V. Write the expression as one logarithm using exponents.
18. $(5 \log_b x + 7 \log_b y) - 3 \log_b z$

VI. Write the logarithmic equation in exponential form.
19. $\log_5 125 = 3$
20. $\ln x = -9$

VII. Write in logarithmic form.
21. $p = 14^t$
22. $16^{5/4} = 32$

VIII. Use the equation of the polynomial function to
a) State the degree
b) State the leading coefficient
c) Describe the end behavior of the graph of the function
23. $f(x) = -20x^3 + 6x - 7$
24. $f(x) = 12x^4 - 3x^3 + 9$
25. $f(x) = -8(x + 2)(x^5 - 2)$
IX. Use the given graph of the polynomial function to
   a) Estimate the x-intercept(s)
   b) State whether the leading coefficient is positive or negative
   c) Determine whether the polynomial function is cubic or quartic

26.

![Graph of polynomial function](image1)

27.

![Graph of polynomial function](image2)

X. Use the graph of the function to find the local maximum and local minimum, round to the nearest hundredth.

28. \( y = x^3 - x^2 - 3x + 2 \)  
29. \( y = 3x^3 - 4x^2 - 6x + 2 \)

XI. Solve the polynomial equation.

30. \( (2x - 7)(x + 5)(x - 5) = 0 \)  
31. \( x^3 - 5x^2 + 6x = 0 \)
32. \( 4x^3 - 24x^2 - x + 6 = 0 \)  
33. \( x^4 - 6x^2 + 8 = 0 \)
34. \( \frac{1}{7} x^3 + 49 = 0 \)
XII. Use the graph of the polynomial function $f(x)$
   a) to solve $f(x) = 0$
   b) find the factorization of $f(x)$

35.  

36.  

XIII. Solve the problem.
37. The future value of $7000$ invested for 5 years at rate $r$, compounded annually, is given by $S = 7000(1 + r)^5$. Find the rate $r$, as a percent, that gives a future value of $9817.86$. Round to the nearest whole percent.

38. A rectangular piece of cardboard measuring 17 inches by 43 inches is to be made into a box with an open top by cutting equal size squares from each corner and folding up the sides. Let $x$ represent the length of a side of each such square. For what value of $x$ will the volume be a maximum? If necessary, round to 2 decimal places.

39. If the price for a product is given by $p = 900 - x^2$, where $x$ is the number of units sold, then the revenue is given by $R = px = 900x - x^3$. How many units must be sold to give zero revenue?

40. Suppose a cost-benefit model is given by $y = \frac{3.4x}{100-x}$, where $y$ is the cost in thousands of dollars for removing $x$ percent of a given pollutant. Find the cost of removing 75% to the nearest dollar.
XIV. Find a) horizontal asymptotes b) vertical asymptotes c) sketch a graph of the function.

41. \( f(x) = \frac{x^2 + x - 9}{x^2 - 9} \)
42. \( f(x) = \frac{x^2 - 4}{x - 2} \)
43. \( f(x) = \frac{x - 3}{x + 4} \)

XV. Solve the equation.

44. \( 1 + \frac{1}{x} = \frac{30}{x^2} \)
45. \( \frac{x + 2}{x + 5} = -3 \)
46. \( \frac{5 - x}{x} + \frac{3}{4} = \frac{7}{x} \)
47. \( \frac{18}{x - 2} = 1 + \frac{20}{x + 2} \)

Answers:

1. The fossil is 7369 years old.
2. a) In 2000, Quickie sold 189 million hamburgers.
   b) In 1996, Quickie sold 117.2 million hamburgers.
3. $7585.67
4. $11,299.12
5. It will take 6.0 years for $9000 to grow to $14,500.
6. It will take 20.1 years to quadruple the investment.
7. a) \( P(t) = 900(1.115^t) \)
   b) In 5.5 days you will have 1638 rabbits.
   c) It will take 19.6 days to have 7621 rabbits
8. a) \( P(t) = 15000 \left(0.5^{t/6}\right) \)
   b) In 3 hours you will have 10,607 bacteria.
   c) It will take 23 hours to reach 1052 bacteria.
9. a) \( f(x) = 1.382(1.457^x), r = 0.996 \)
   b) \( g(x) = 1.119 + 3.928\ln x, r = 0.938 \)
   c) The exponential since its r value is closest to 1.
   d) \( f(7) = 1.382(1.457^7) = 19.3 \)
10. \( x \approx 7.887 \)
11. \( x \approx 4.953 \)
12. \( x = 11 \)
13. \( x = 8 \)
14. \( x = \sqrt{86} \approx 9.274 \)
15. \( \log_{178} \approx 3.1430 \)
16. \( \log_{0.435} \approx -0.7577 \)
17. \( \log_a 7 + 3 \log_a x + \log_a y + 4 \log_a z \)
18. \( \log_b \left(\frac{x^5 y^7}{z^3}\right) \).
19. \( 5^3 = 125 \)
20. \( x = e^{-9} \)
21. \( \log_{14} p = t \)
22. \( \log_{16} 32 = \frac{5}{4} \)
23. a) 3  b) −20  c) Left end rises and right end falls
24. a) 4  b) 12  c) Both ends rise
25. a) 6  b) −8  c) Both ends fall
26. a) (−2,0), (1,0), (3,0)  b) positive  c) cubic
27. a) (−3,0), (−1,0), (0,0), (2,0)  b) negative  c) quartic
28. Local Max = (−0.72,3.27), Local Min = (1.39,−1.42)
29. Local Max = (−0.49,3.63), Local Min = (1.37,−6.01)
30. \( x = \frac{7}{2}, −5, 5 \)
31. \( x = 0, 2, 3 \)
32. \( x = \frac{1}{2}, −\frac{1}{2}, 6 \)
33. \( x = ±2, ±\sqrt{2} \)
34. \( x = −7 \)
35. a) \( x = −6, −2, 4 \)  b) \( (x + 6)(x + 2)(x − 4) \)
36. a) \( x = −3, 0, 2 \)  b) \( (x + 3)^2(x)^2(x − 2)^2 \)
37. 7%
38. 3.75 in
39. \( x = 0, 30 \)
40. $10,200
41. a) \( y = 1 \)  b) \( x = ±3 \)  c) use calculator to check graph
42. a) none  b) none  c) use calculator to check graph, hole at \( x = 2 \)
43. a) \( y = 1 \)  b) \( x = −4 \)  c) use calculator to check graph
44. \( x = −6, 5 \)
45. \( x = −\frac{17}{4} \)
46. \( x = −8 \)
47. \( x = 8, −10 \)