NOTES: sample means sampling distributions

A Simple Random Sample used to obtain $\bar{x}$ provides an unbiased estimator of $\mu$. In other words, the mean of the sampling distribution of the $\bar{x}$ values is $\mu$. In notation:

$$\mu(\bar{x}) = \mu$$

Also, the standard deviation of the sampling distribution of the $\bar{x}$ values is given by

$$SD(\bar{x}) = \frac{\sigma}{\sqrt{n}}$$

where $\sigma$ is the standard deviation for $x$ and $n$ is the sample size. For each fixed sample size $n$ there is a corresponding sample mean ($\bar{x}$) distribution.

Conditions/Requirements:
When is the sampling distribution of $\bar{x}$ normal? There are two ways this can happen.

If you have
- random samples (SRS)
- $x$ has a normal distribution
then $\bar{x}$ has a normal distribution.

CENTRAL LIMIT THEOREM

If you have
- random samples (SRS)
- sample size sufficiently large ($n \geq 30$)
- finite population standard deviation ($\sigma$ is a number)
then the sample means ($\bar{x}$ values) have an approximately normal distribution. The greater the sample size, the closer the corresponding $\bar{x}$ distribution resembles a normal distribution.

EXERCISES:

1. Vehicle speeds at a certain highway location are believed to have approximately a normal distribution with mean 60 mph and standard deviation 6 mph. The speeds for a randomly selected sample of 36 vehicles will be recorded.
   a. Explain why the sampling distribution for sample means has a normal distribution. Give numerical values for the mean and standard deviation of the sampling distribution of possible sample means for randomly selected samples of 36 from the population of vehicle speeds.
Since we have an SRS “…randomly selected sample of 36 vehicles…” and $x$ (vehicle speeds) has an approximate normal distribution, then $\overline{x}$ (sample means) has an approximate normal distribution.

Another way we can verify that $\overline{x}$ has an approximate normal distribution here is by noting that an SRS “…randomly selected sample of 36 vehicles…” $n = 36 \geq 30$ $\sigma = 6$ is finite. So it follows by the Central Limit Theorem as well.

Finally, we have $\mu(\overline{x}) = \mu = 60$ 

$$SD(\overline{x}) = \frac{\sigma}{\sqrt{n}} = \frac{6}{\sqrt{36}} = 1$$

b. Use the Empirical Rule to find values that fill in the blanks in the following sentence. For a random sample of 36 vehicles, there is a 95% chance that mean vehicle speed in the sample will be between ________ and ________ mph.

**ANSWER:**

There is a 95% chance that mean vehicle speed in the sample will be between 58 mph and 62 mph.

![Bell Curve Diagram](image_url)

57 58 59 60 61 62 63 $\overline{x}$

57 58 59 60 61 62 63 $\overline{x}$

c. Sample speeds for a random sample of 36 vehicles are measured at this location, and the sample mean is 57.2 mph. What is the $z$-score which corresponds with 57.2?

**ANSWER:**

$$z = \frac{(\overline{x} - \mu)}{\frac{\sigma}{\sqrt{n}}} = \frac{(57.2 - 60)}{\frac{6}{\sqrt{36}}} = \frac{-2.8}{1} = -2.8$$
The scores of individual students on the American College Testing (ACT) composite college entrance examination have a normal distribution with mean 18.6 and standard deviation 5.9.

a. What is the probability that a single student randomly chosen from all those taking the test, scores 21 or higher?

**ANSWER:**

\[
\text{normalcdf}(21, 1E99, 18.6, 5.9) = 0.342
\]

b. Now take a random sample of 50 students who took the test. What are the mean and standard deviation of the distribution of the average score for 50 students? Why is the sample mean distribution normal?

**ANSWER:**

We have

\[
\mu(\bar{x}) = \mu = 18.6
\]

\[
SD(\bar{x}) = \frac{\sigma}{\sqrt{n}} = \frac{5.9}{\sqrt{50}} \approx 0.8344
\]

Since the 50 students are picked randomly, and \( x \) (ACT scores) has a normal distribution, then \( \bar{x} \) (sample means) has a normal distribution.

In addition, we know the following:
- SRS “…a random sample of 50 students…”
- \( n = 50 \geq 30 \)
- \( \sigma = 5.9 \) is finite

So another reason \( \bar{x} \) has a normal distribution is by the Central Limit Theorem.

c. What is the probability that the mean score of these 50 students is 21 or higher?

**ANSWER:**

\[
\text{normalcdf}(21, 1E99, 18.6, \frac{5.9}{\sqrt{50}}) = 0.002
\]