NOTES: sample proportions sampling distributions

A Simple Random Sample used to obtain \( \hat{p} \) provides an unbiased estimator of \( p \). In other words, the mean of the sampling distribution of the \( \hat{p} \) numbers is \( p \). In notation:

\[
\mu(\hat{p}) = p
\]

Also, the standard deviation of the sampling distribution of the \( \hat{p} \) numbers is given by (where \( n \) is the sample size):

\[
SD(\hat{p}) = \sqrt{\frac{p(1-p)}{n}}
\]

Conditions/Requirements:

Sample proportions (\( \hat{p} \) numbers) have a normal sampling distribution, with mean and standard deviation formulas above, provided certain conditions are met:

1. The sample proportion \( \hat{p} \) must be obtained from a Simple Random Sample (SRS).
2. \( np \geq 10 \). (Note that some books say that it is sufficient for \( np \geq 5 \).)
3. \( n(1-p) \geq 10 \). (Note that some books say that it is sufficient for \( n(1-p) \geq 5 \).)
4. The population size is at least ten times the sample size (\( n \)).

EXERCISES:

1. Suppose medical researchers think that 0.70 is the proportion of all teenagers with high blood pressure whose blood pressure would decrease if they took calcium supplements. To test this theory, the researchers plan a clinical trial (experiment) in which 200 random teenagers with high blood pressure will take regular calcium supplements.
   a. Assume 0.70 actually is the population proportion that would experience a decrease in blood pressure. What are the numerical values of the mean and standard deviation of the sampling distribution of the sample proportions for samples of 200 teenagers?

   b. Use the results of part (a) to calculate an interval that will contain the sample proportion for about 99.7% of all samples of 200 teenagers.

   c. In the clinical trial, 120 of the 200 teenagers taking calcium supplements experienced a decrease in blood pressure. What is the value of \( \hat{p} \) for this sample? Is this value a parameter or a statistic?

   d. What is the probability of having 120 or fewer experience a decrease of blood pressure out of a sample of 200 teenagers taking calcium supplements?

   e. We have used the properties of a normal distribution. Show that the conditions/requirements are satisfied.
2. **DO YOU DRINK THE CEREAL MILK?** A *USA Today* poll asked a random sample of 1012 U.S. adults what they do with the milk in the bowl after they have eaten the cereal. Of the respondents, 67% said that they drink it. Suppose that 70% of U.S. adults actually drink the cereal milk.

   a. Explain why you may use the formulas for normal distributions of sample proportions.

   b. Find the probability of obtaining a sample of 1012 adults in which 67% or more say they drink the cereal milk.

   c. What proportion would say that they drink the cereal milk if it is in the lower 20% of all samples of 1012 U.S. adults?

   d. What is the z-score for a sample of 1012 U.S. adults in which 75% of them said that they drink the cereal milk?