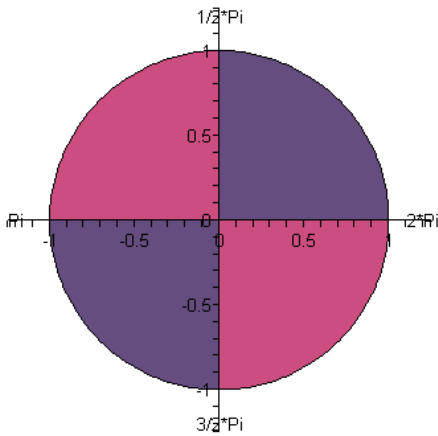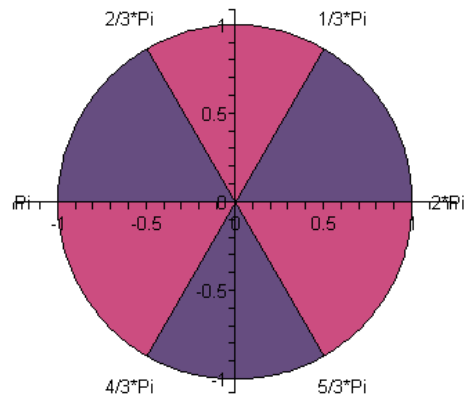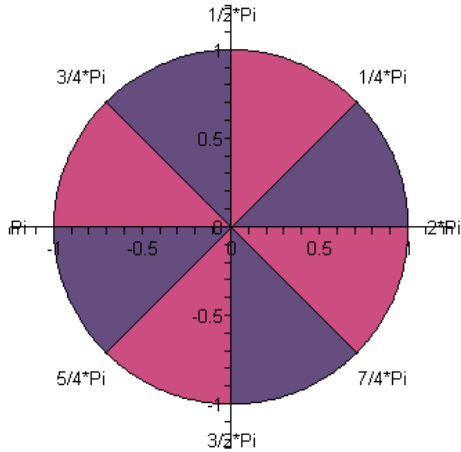# Angle Spectrum and Reference Angles

To locate Special Angles in radian measure, you do not need to change the angles to degree measure. All you need to do is to subdivide the circle in multiples of $\pi/2$, multiples of $\pi/3$, multiples of $\pi/4$, and multiples of $\pi/6$, as shown below. Remember that the radian measure of a straight angle (half of the circle) is π, and the radian measure of a revolution of the initial side of the angle (the full circle) is 2π.
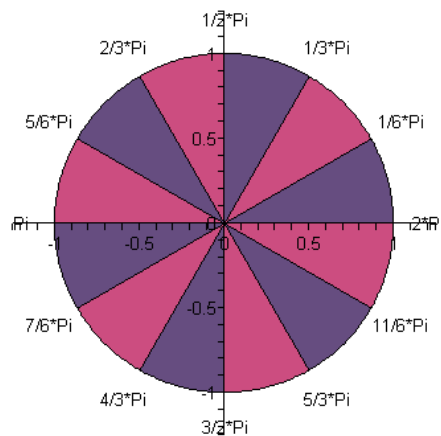
Spectrum of $\pi/2$
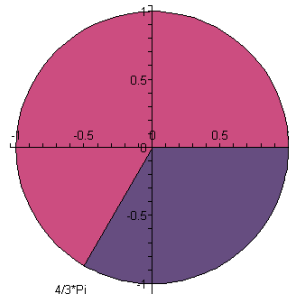
Spectrum of $\pi/3$

Spectrum of $\pi/4$

Spectrum of $\pi/6$

Example:
Locate the angle $4\pi/3$ and find its reference angle.
We see from the second graph above that $4\pi/3$ is in quadrant III and its reference angle is $\pi/3$. See the graph below.
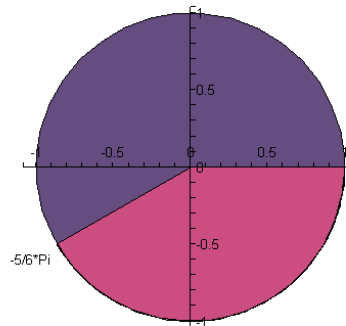

To find the reference angle of $4\pi/3$ without using the graphs, we notice that $4\pi/3 = 3\pi/3 + \pi/3$. Since $3\pi/3 = \pi$, $4\pi/3$ is $\pi/3$ more than π, so the reference angle is $\pi/3$.

Example:
Locate the angle $-5\pi/6$ and find its reference angle.
We see from the last graph above that $-5\pi/6$ is coterminal with $7\pi/6$, so the angle is in quadrant III and its reference angle is $\pi/6$. See the graph below.


To find the reference angle of $-5\pi/6$ without using the graphs, we notice that $-5\pi/6 = -6\pi/6 + \pi/6$. Since $-6\pi/6 = -\pi$, $-5\pi/6$ is $\pi/6$ more than $-\pi$, so the reference angle is $\pi/6$.

Another way of finding reference angles is making a rough sketch of you standard angle if the appropriate quadrant and sketching the reference angle.
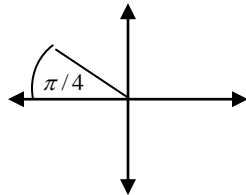
Example:
Find the reference angle of $7\pi/4$.
Since $7\pi/4$ is in quadrant IV, $7\pi/4$ is $\pi/4$ units away from $8\pi/4 = 2\pi$. So the reference angle is $\pi/4$.
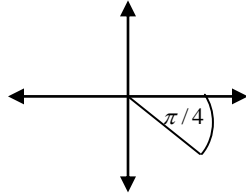
Example:
Find the reference angle of $3\pi/4$.
Since $3\pi/4$ is in quadrant II, $3\pi/4$ is $\pi/4$ units away from $4\pi/4 = \pi$. So the reference angle is $\pi/4$.

Example:
Find the reference angle of $-7\pi/6$.
Since $-7\pi/6$ is in quadrant II, $-7\pi/6$ is $\pi/6$ units away from $-6\pi/6 = -\pi$. So the reference angle is $\pi/6$.