1. Find the exact value of $\sin (2 x), \cos (2 x), \tan (2 x), \sin \frac{x}{2}, \cos \frac{x}{2}, \tan \frac{x}{2}$.
a) $\sec x=5$ with $\frac{3 \pi}{2}<x<2 \pi$
b) $\cot x=-5$ with $x$ in Quadrant II
2. Find the exact value of $\sin$, cos and tan for the given angles:
a) $105^{\circ}$
b) $-75^{\circ}$
c) $345^{\circ}$
3. Use the fundamental identities to find the exact values of the remaining trigonometric functions of $\boldsymbol{x}$.
a) $\cos x=\frac{-4}{9}$ and $\sin x>0$
b) $\sec x=\frac{-61}{60}$ and $\tan x>0$
4. Find the exact values for the $\sin (x+y), \cos (x+y), \tan (x+y), \sin (x-y), \cos (x-y)$, $\tan (x-y)$, and state which quadrant $x+y$ and $x-y$ is in, given the following conditions:
a) $\sin x=\frac{2}{3}, \cos y=\frac{-1}{4}$, x in quadrant II, and y in quadrant III
b) $\cos x=\frac{-1}{3}$, tany $=\frac{1}{2}, \mathrm{x}$ in quadrant II, and y in quadrant III
c) $\tan x=\frac{3}{4}, \tan y=\frac{-1}{2}, x$ in quadrant III, and y in quadrant IV
5. Use a graphing calculator to test whether the following is an identity. If it is an identity, verify it. If it is not an identity, find a value of $\mathbf{x}$ for which both sides are defined but not equal.
a) $\frac{\cos (-x)}{\sin x \cot (-x)}=1$
b) $\frac{\tan x}{\sin x+2 \tan x}=\frac{1}{\cos x-2}$
c) $\frac{\cos x}{1-\sin x}+\frac{\cos x}{1+\sin x}=2 \sec x$
6. Verify the following identities. You will have options of identities to verify.
a) $\quad \sin ^{2} x-\sin ^{2} y=\cos ^{2} y-\cos ^{2} x$
b) $2 \cos ^{3} x-\cos x=\frac{\cos ^{2} x-\sin ^{2} x}{\sec x}$
c) $2 \cos A-\sec A=\cos A-\frac{\tan A}{\csc A}$
d) $\frac{\cos a}{\sec a}+\frac{\sin a}{\csc a}=\sec ^{2} a-\tan ^{2} a$
e) $\frac{\csc \theta+\cot \theta}{\tan \theta+\sin \theta}=\cot \theta \csc \theta$
7. Evaluate exactly using an appropriate identity.
a) $\sin 165^{\circ} \sin 15^{\circ}$
b) $\cos 165^{\circ}-\cos 75^{\circ}$

## Trigonometry Test 4 Practice, Chapter 4

## ANSWERS:

1. a) $\sin (2 x)=\frac{-4 \sqrt{6}}{25}, \cos (2 x)=\frac{-23}{25}, \tan (2 x)=\frac{4 \sqrt{6}}{23}, \sin \frac{x}{2}=\sqrt{\frac{2}{5}}, \cos \frac{x}{2}=-\sqrt{\frac{3}{5}}$,
$\tan \frac{x}{2}=\frac{-\sqrt{6}}{3}$ or $-\sqrt{\frac{2}{3}}$
b) $\sin (2 x)=\frac{-5}{13}, \cos (2 x)=\frac{12}{13}, \tan (2 x)=\frac{-5}{12}, \sin \frac{x}{2}=\sqrt{\frac{5+\sqrt{26}}{2 \sqrt{26}}}, \cos \frac{x}{2}=\sqrt{\frac{-5+\sqrt{26}}{2 \sqrt{26}}}$,

$$
\tan \frac{x}{2}=\frac{1}{-5+\sqrt{26}}
$$

2. a) $\sin =\frac{1+\sqrt{3}}{2 \sqrt{2}}$ or $\frac{\sqrt{2}+\sqrt{6}}{4}, \cos =\frac{1-\sqrt{3}}{2 \sqrt{2}}$ or $\frac{\sqrt{2}-\sqrt{6}}{4}$, $\tan =\frac{1+\sqrt{3}}{1-\sqrt{3}}$
b) $\sin =\frac{-\sqrt{3}-1}{2 \sqrt{2}}$ or $\frac{-\sqrt{6}-\sqrt{2}}{4}$, $\cos =\frac{\sqrt{3}-1}{2 \sqrt{2}}$ or $\frac{\sqrt{6}-\sqrt{2}}{4}$, tan $=\frac{-\sqrt{3}-1}{\sqrt{3}-1}$ or $\frac{-1-\sqrt{3}}{-1+\sqrt{3}}$
c) $\sin =\frac{-\sqrt{3}+1}{2 \sqrt{2}}$ or $\frac{-\sqrt{6}+\sqrt{2}}{4}, \cos =\frac{1+\sqrt{3}}{2 \sqrt{2}}$ or $\frac{\sqrt{2}+\sqrt{6}}{4}, \tan =\frac{1-\sqrt{3}}{1+\sqrt{3}}$
3. a) $\sin x=\frac{\sqrt{65}}{9}, \tan x=\frac{-\sqrt{65}}{4}, \csc x=\frac{9}{\sqrt{65}}$ or $\frac{9 \sqrt{65}}{65}, \sec x=\frac{-9}{4}, \cot x=\frac{-4}{\sqrt{65}}$ or $\frac{-4 \sqrt{65}}{65}$
b) $\sin x=\frac{-11}{61}, \cos x=\frac{-60}{61}, \tan x=\frac{11}{60}, \csc x=\frac{-61}{11}, \cot x=\frac{60}{11}$
4. a) $\sin (x+y)=\frac{-2+5 \sqrt{3}}{12}, \cos (x+y)=\frac{\sqrt{5}+2 \sqrt{15}}{12}, \tan (x+y)=\frac{-2+5 \sqrt{3}}{\sqrt{5}+2 \sqrt{15}}$ or $\frac{32 \sqrt{5}-9 \sqrt{15}}{55},(x+$ $y)$ in quad I
$\sin (x-y)=\frac{-2-5 \sqrt{3}}{12}, \cos (x-y)=\frac{\sqrt{5}-2 \sqrt{15}}{12}$,
$\tan (x-y)=\frac{-2-5 \sqrt{3}}{\sqrt{5}-2 \sqrt{15}}$ or $\frac{32 \sqrt{5}+9 \sqrt{15}}{55},(x-y)$ in quad III
b) $\sin (x+y)=\frac{-4 \sqrt{2}+1}{3 \sqrt{5}}$ or $\frac{-4 \sqrt{10}+\sqrt{5}}{15}, \cos (x+y)=\frac{2+2 \sqrt{2}}{3 \sqrt{5}}$ or $\frac{2 \sqrt{5}+2 \sqrt{10}}{15}$,

$$
\tan (x+y)=\frac{-4 \sqrt{2}+1}{2+2 \sqrt{2}} \text { or } \frac{5 \sqrt{2}-9}{2},(x+y) \text { in quad IV }
$$

$\sin (x-y)=\frac{-4 \sqrt{10}-\sqrt{5}}{15}, \cos (x-y)=\frac{2 \sqrt{5}-2 \sqrt{10}}{15}, \tan (x-y)$

$$
=\frac{-4 \sqrt{2}-1}{2-2 \sqrt{2}} \text { or } \frac{5 \sqrt{2}+9}{2},(x-y) \text { in quad III }
$$

c) $\sin (x+y)=\frac{-2}{5 \sqrt{5}}$ or $\frac{-2 \sqrt{5}}{25}, \cos (x+y)=\frac{-11}{5 \sqrt{5}}$ or $\frac{-11 \sqrt{5}}{25}, \tan (x+y)=\frac{2}{11},(x+$
$y$ )in quad III

$$
\sin (x-y)=\frac{-2}{\sqrt{5}} \text { or } \frac{-2 \sqrt{5}}{5}, \cos (x-y)=\frac{-1}{\sqrt{5}} \text { or } \frac{-\sqrt{5}}{5}, \tan (x-y)=2,(x-y) \text { in quad III }
$$

5. a) Not an Identity, $x=\frac{\pi}{4}$, answers may vary.
b) Not an Identity, $x=\frac{\pi}{4}$, answers may vary.
c) Identity
6. 

a) $\frac{-\sqrt{3}}{4}+\frac{1}{2}$ or $\frac{1}{2}-\frac{\sqrt{3}}{4}$
b) $\frac{-\sqrt{3}}{\sqrt{2}}$ or $\frac{-\sqrt{6}}{2}$

