1. Fill in the blanks in the following table using exact values.

θ	Reference Angle	$\sin\theta$	$\cos\theta$	$\tan\theta$
$\frac{11\pi}{6}$				
$225°$				

2. Find the **exact** values of *x* that satisfy the given condition.

   a) $\cos x = -1, 0 \le x \le 6\pi$ b) $\cos x = 0, -\pi \le x \le 2\pi$

3. Find the **exact** value of the following.

   a) $\sin\left(\frac{7\pi}{2}\right)$ b) $\cos\left(\frac{-3\pi}{2}\right)$ c) $cos\left(\frac{-3\pi}{4}\right)$

4. Use the fundamental identities to find the **exact** value of $\sin x, \cos x, \tan x, \csc x, \cot x, \sin(2x), \cos(2x), \tan(2x), sin\frac{x}{2}, cos\frac{x}{2}, tan\frac{x}{2}$, given that $secx = \frac{-5}{3}$ and $sin\, x < 0$.

Find the amplitude, period, and phase shift, and vertical translation of the following:

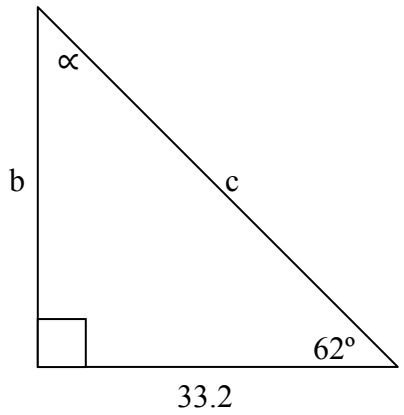
5. $y = 4 - 2sin\left(4x + \frac{\pi}{6}\right)$
6. $y = -5 - 3sin\left(6x - \frac{\pi}{2}\right)$

7. What is the maximum value of $y = 10 - 4\cos\left(7\pi x + \frac{\pi}{3}\right)$

8. What is the minimum value of $y = -9 + 4\cos\left(8\pi x + \frac{\pi}{3}\right)$.

9. Find the x-intercepts of the trigonometric functions:
   a) $f(x) = 2sin^2 x + \sin(2x)$ for $0 \le x < 2\pi$ give exact answer in terms of π.
   b) $f(x) = \sin(2x) - \sqrt{3}sinx$ for $0 \le x < 2\pi$ give exact answer in terms of π.
   c) $f(x) = 2sin^2\theta + 5cos\theta + 1$ for $0° \le \theta < 360°$.

10. Solve the right triangle below.



11. Assuming each angle given is in standard position; find the quadrant of its terminal side.

a) 842º b) $\frac{-12\pi}{5}$

12. Suppose $\theta = 42°$ is a central angle in a circle with radius of 14.2 meters. Recall that $s = r\theta$ (angle in radians) and $A = \frac{1}{2}r^2\theta$ (angle in radians)

a) Find the length of the arc subtended by θ.
b) Find the area of the circular sector formed by θ.

13. Use a calculator to find each to four decimal places.

a) $\sec(-4.578)$ b) $\sin(167.5°)$

14. Consider the equation $y = -3 + 2\cos\left(\pi x - \frac{\pi}{2}\right)$.

a) What is the amplitude?
b) What is the period?
c) What is the phase shift?
d) What is the vertical translation?
e) Graph $y = -3 + 2\cos\left(\pi x - \frac{\pi}{2}\right)$ from $-2 \le x \le 2$.
f) What is the domain?
g) What is the range?

15. If $\sin(x) = \frac{-\sqrt{3}}{2}$ and $\cos(x) = \frac{-1}{2}$, find $\csc(x)$ and $\cot(x)$.

16. Find the **exact** value of each of the other five trigonometric functions for an angle θ given:

a) $\csc(\theta) = \frac{-\sqrt{10}}{3}$ and $\cos(\theta) > 0$ b) $\cos(\theta) = \frac{-4}{9}$ and $\sin(\theta) > 0$

17. Find the **exact** values for the sin(x + y), cos(x + y), tan(x + y), sin(x – y), cos(x – y), tan(x – y), and state which quadrant x + y and x – y is in, given the following conditions:

    a) $sinx = \frac{2}{3}, cosy = \frac{-1}{4}$, x in quadrant II, and y in quadrant III

    b) $cosx = \frac{-1}{3}, tany = \frac{1}{2}$, x in quadrant II, and y in quadrant III

    c) $tanx = \frac{3}{4}, tany = \frac{-1}{2}$, x in quadrant III, and y in quadrant IV

18. Verify the following identities. You will have options of identities to verify.

    a) $\sin^2 x - \sin^2 y = \cos^2 y - \cos^2 x$ b) $2\cos^3 x - \cos x = \frac{\cos^2 x - \sin^2 x}{\sec x}$

    c) $2cosA - secA = cosA - \frac{tanA}{cscA}$ d) $\frac{\cos a}{\sec a} + \frac{\sin a}{\csc a} = \sec^2 a - \tan^2 a$

    e) $\frac{\csc\theta + \cot\theta}{\tan\theta + \sin\theta} = \cot\theta \csc\theta$

19. Use appropriate identities to find $\cos 75°$ **exactly**.
20. Find four solutions of each equation.
    a) $\sin(3x + 2) = -1$ b) $\cos(\pi x + 5) = 1$
    c) $6\sin(2x) + 9 = 3$

21. Find two solutions in different quadrants of each equation.
    a) $3\cos(5x) = -2$ b) $5\sin(\pi x + 5) + 6 = 7$

22. Solve the triangle with $a = 28, b = 32, c = 22$.

23. Solve the triangle with $\propto = 58°$, $\gamma = 12°$, and $a = 10\ cm$.

24. The current at a point in an electric circuit is $C(t) = 20\sin(30\pi t)$ for $t \geq 0$. Find the first time when the current is 15. Round to 3 decimal places.

25. Suppose that a ladder that is 12 feet long leans against a house and the base of the ladder is 4 feet from the wall. What angle does the ladder make with the ground? Give the answer rounded to the nearest degree.

26. A ramp to provide wheelchair access to a building is 20 feet long and it rises up to a doorway that is 2 feet above the ground. What angle does the ramp make with the ground? Give the answer rounded to the nearest degree.

27. A boat leaves a dock and travels 6 miles due east of the dock and then turns and travels 3 miles north. A boat captain wants to tell a friend at the dock how to travel directly to the boat. How far is the boat from the dock and what angle does the direct line from the dock to the boat make with the east direction?

28. The line with the equation $y = 5x$ makes what angle with the positive x-axis?

29. The line with the equation $y = -3x$ makes what positive angle with the positive x-axis?

30. Draw a right triangle and label the sides so that $\sin(\theta) = \frac{4}{7}$. What is the exact value of the angle θ? What is the $\cos(\theta)$?

31. What angle with the positive x-axis does the line from the origin to the point $(3,5)$ make? Give the exact value and the decimal approximation rounded to two decimal places.

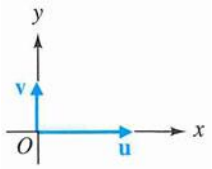
32. What positive angle with the positive x-axis does the line from the origin to the point $(-2,7)$ make? Give the exact value and the decimal approximation rounded to two decimal places.

33. Find the measure in decimal degrees, rounded to the nearest hundredths, of a central angle subtended by a chord of length 112 ft in a circle of radius 72.8 ft.

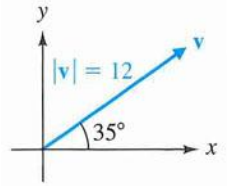
34. Find the perimeter (to the nearest centimeter) of a regular heptagon (seven-sided figure) inscribed in a circle with radius 5 cm.

35. Evaluate exactly using an appropriate identity.
a) $sin165°sin15°$ b) $cos165° - cos75°$

36. Two vectors **u** and **v** are located in a coordinate system, as indicated in the figure. Find the direction (relative to the *x* axis) and magnitude of **u** + **v** if $|\boldsymbol{u}| = 8.0$ and $|\boldsymbol{v}| = 5.0$.

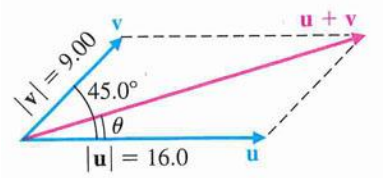


37. Find the magnitude of the horizontal and vertical components of the vector **v** located in a coordinate system as indicated in the figure, then write **v** in terms of the unit vectors **i** and **j**.



38. Given $A = (-3,2) and\ B = (-1,-3)$ represent the geometric vector $\overrightarrow{AB}$ as an algebraic vector $\langle a, b\rangle$.
39. Find the magnitude of the vector $\langle -5,12\rangle$.
40. $\langle 2,-1\rangle \cdot \langle -3,2\rangle =$
41. $(2\boldsymbol{i} + \boldsymbol{j}) \cdot (3\boldsymbol{i} - 2\boldsymbol{j}) =$
42. For each pair of vectors, are they orthogonal, parallel, or neither?
(A) $\langle 4,-3\rangle\ and\ \langle 8,6\rangle$ (B) $\langle \frac{5}{2}, \frac{-1}{2}\rangle\ and\ \langle -10,2\rangle$ (C) $\langle 10,-6\rangle\ and\ \langle 3,5\rangle$

43. Given the vector diagram, find $|\boldsymbol{u}+\boldsymbol{v}|$ and θ.



44. Find a unit vector **u** with the same direction as $\boldsymbol{v} = \langle -8,15\rangle$.
45. Express **v** in terms of **i** and **j** unit vectors.
    (A) $\boldsymbol{v} = \langle -5,7\rangle$ (B) $\boldsymbol{v} = \langle 0,-3\rangle$ (C) $\boldsymbol{v} = \overrightarrow{AB}; A = (4,-2); B = (0,-3)$
46. Determine which vector pairs are orthogonal using properties of the dot product.
    (A) $\boldsymbol{u} = \langle -12,3\rangle; \boldsymbol{v} = \langle 2,8\rangle$ (B) $\boldsymbol{u} = -4\boldsymbol{i} + \boldsymbol{j}; \boldsymbol{v} = -\boldsymbol{i} + 4\boldsymbol{j}$

For the following problems, find: (A) **u + v** (B) **u – v** (C) 3**u** – 2**v** (D) 2**u** – 3**v** + **w**

47. $\boldsymbol{u} = \langle 4,0\rangle; \boldsymbol{v} = \langle -2,-3\rangle; \boldsymbol{w} = \langle 1,-1\rangle$
48. $\boldsymbol{u} = 3\boldsymbol{i} - \boldsymbol{j}; \boldsymbol{v} = 2\boldsymbol{i} - 3\boldsymbol{j}; \ \boldsymbol{w} = -2\boldsymbol{j}$

Plot in a polar coordinate system:

49. $A = (5,-30°); B = (4,-45°); C = (9,-90°)$
50. $A = \left(-6,\frac{\pi}{6}\right); B = \left(-5,\frac{\pi}{2}\right); C = \left(-8,\frac{\pi}{4}\right)$

51. Change the given polar coordinates to exact rectangular coordinates:
    a) $(-3,90°)$ b) $\left(10,\frac{\pi}{4}\right)$
52. Change the given rectangular coordinates to exact polar coordinates:
    a) $(0,-12)$ b) $(-16,0)$
53. Change to polar form: a) $y^2 = 5y - x^2$ b) $y = 8$
54. Graph in Polar Form, then change to rectangular form:
    a) $r = 2cos\theta + 3sin\theta$ b) $r = 3cos\theta$

ANSWERS:

1.

θ	Reference Angle	$\sin\theta$	$\cos\theta$	$\tan\theta$
$\frac{11\pi}{6}$	$\frac{\pi}{6}$	$\frac{-1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{-1}{\sqrt{3}}$
$225°$	$45°$	$\frac{-1}{\sqrt{2}}$	$\frac{-1}{\sqrt{2}}$	1

2. a) $\pi, 3\pi, 5\pi$ b) $\frac{-\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}$
3. a) -1 b) 0 c) $\frac{-1}{\sqrt{2}}$
4. $sinx = \frac{-4}{5}, cosx = \frac{-3}{5}, tanx = \frac{4}{3}, cscx = \frac{-5}{4}, cotx = \frac{3}{4}, \sin(2x) = \frac{24}{25},$
   $\cos(2x) = \frac{-7}{25}, \tan(2x) = \frac{-24}{7}, sin\frac{x}{2} = \frac{2}{\sqrt{5}}, cos\frac{x}{2} = \frac{-1}{\sqrt{5}}, tan\frac{x}{2} = -2$
5. $amp = 2, per = \frac{\pi}{2}, horizontal\ shift = \frac{-\pi}{24}, vertical\ translation = up\ 4$
6. $amp = 3, per = \frac{\pi}{3}, horizontal\ shift = \frac{\pi}{12}, vertical\ translation = down\ 5$

7. 14
8. -13
9. a) $x = \frac{3\pi}{4}, \pi, \frac{7\pi}{4}$ b) $x = 0, \frac{\pi}{6}, \pi, \frac{11\pi}{6}$ c) $\theta = 120°, 240°$
10. $b = 62.4, c = 70.7, \propto = 28°$
11. a) II b) IV
12. a) 10.4 m b) 73.9 sq m
13. a) -7.4635 b) 0.2164
14. a) 2 b) 2 c) $\frac{1}{2}$ d) -3 e) use calculator to check work f) all reals g) $-5 \le y \le -1$
15. $\csc(x) = \frac{-2}{\sqrt{3}}, \cot(x) = \frac{1}{\sqrt{3}}$
16. a) $\sin(\theta) = \frac{-3}{\sqrt{10}}, \cos(\theta) = \frac{1}{\sqrt{10}}, \tan(\theta) = -3, \csc(\theta) = \frac{-\sqrt{10}}{3}, \sec(\theta) = \sqrt{10}, \cot(\theta) = \frac{-1}{3}$
b) $\sin(\theta) = \frac{\sqrt{65}}{9}, \cos(\theta) = \frac{-4}{9}, \tan(\theta) = \frac{-\sqrt{65}}{4}, \csc(\theta) = \frac{9\sqrt{65}}{65}, \sec(\theta) = \frac{-9}{4}, \cot(\theta) = \frac{-4\sqrt{65}}{65}$
1. 17. a)

$\sin(x+y) = \frac{-2+5\sqrt{3}}{12}, \cos(x+y) = \frac{\sqrt{5}+2\sqrt{15}}{12}, \tan(x+y) = \frac{-2+5\sqrt{3}}{\sqrt{5}+2\sqrt{15}}$ *or* $\frac{32\sqrt{5}-9\sqrt{15}}{55}, (x+y)$in quad I

$\sin(x-y) = \frac{-2-5\sqrt{3}}{12}, \cos(x-y) = \frac{\sqrt{5}-2\sqrt{15}}{12},$

$\tan(x-y) = \frac{-2-5\sqrt{3}}{\sqrt{5}-2\sqrt{15}}$ *or* $\frac{32\sqrt{5}+9\sqrt{15}}{55}, (x-y)$in quad III

b) $\sin(x+y) = \frac{-4\sqrt{2}+1}{3\sqrt{5}}$ *or* $\frac{-4\sqrt{10}+\sqrt{5}}{15}, \cos(x+y) = \frac{2+2\sqrt{2}}{3\sqrt{5}}$ *or* $\frac{2\sqrt{5}+2\sqrt{10}}{15},$

$\tan(x+y) = \frac{-4\sqrt{2}+1}{2+2\sqrt{2}}$ *or* $\frac{5\sqrt{2}-9}{2}, (x+y)$in quad IV

$\sin(x-y) = \frac{-4\sqrt{10}-\sqrt{5}}{15}, \cos(x-y) = \frac{2\sqrt{5}-2\sqrt{10}}{15}, \tan(x-y)$

$= \frac{-4\sqrt{2}-1}{2-2\sqrt{2}}$ *or* $\frac{5\sqrt{2}+9}{2}, (x-y)$in quad III

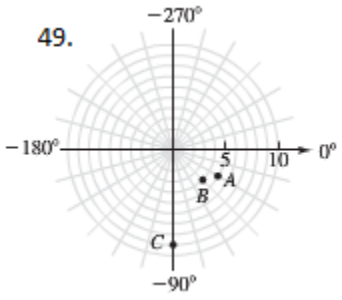
c) $\sin(x+y) = \frac{-2}{5\sqrt{5}}$ *or* $\frac{-2\sqrt{5}}{25}, \cos(x+y) = \frac{-11}{5\sqrt{5}}$ *or* $\frac{-11\sqrt{5}}{25}, \tan(x+y) = \frac{2}{11}, (x+y)$in quad III

$\sin(x-y) = \frac{-2}{\sqrt{5}}$ *or* $\frac{-2\sqrt{5}}{5}, \cos(x-y) = \frac{-1}{\sqrt{5}}$ *or* $\frac{-\sqrt{5}}{5}, \tan(x-y) = 2, (x-y)$in quad III19. $\frac{\sqrt{3}-1}{2\sqrt{2}} = \frac{\sqrt{6}-\sqrt{2}}{4}$

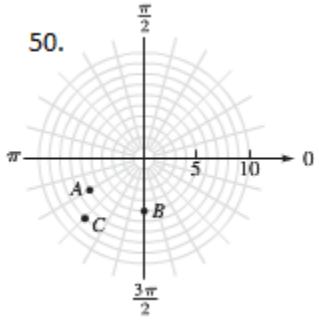
20. a) $x = \frac{-5\pi-4}{6}, x = \frac{-\pi-4}{6}, x = \frac{3\pi-4}{6}, x = \frac{7\pi-4}{6}$
b) $x = \frac{-2\pi-5}{\pi}, x = -\frac{5}{\pi}, x = \frac{2\pi-5}{\pi}, x = \frac{4\pi-5}{\pi}$
c) $x = \frac{-5\pi}{4}, \frac{-\pi}{4}, \frac{3\pi}{4}, \frac{7\pi}{4}$
21. a) $x_1 = 0.4601, \ x_2 = 0.7965$ b) $x_1 = -1.5274, \ x_2 = -0.6557$
22. $\propto = 59°, \beta = 79°, \gamma = 42°$
23. $\beta = 110°, \ b = 11.08\ cm, \ c = 2.45\ cm$
24.. $x = 0.009$
25. $\theta = 71°$

26. $\theta = 6°$
27. $d = 6.7\ miles, \theta = 27°$
28. $\theta = 79°$
29. $\theta = 108°$
30. $\theta = \sin^{-1}\left(\frac{4}{7}\right), \cos\theta = \frac{\sqrt{33}}{7}$
31. $\theta = \tan^{-1}\left(\frac{5}{3}\right), \theta = 59.04°$
32. $\theta = \tan^{-1}\left(\frac{-7}{2}\right), \theta = 105.95°$
33. $100.57°$
34. 30 cm
35. a) $\frac{-\sqrt{3}}{4} + \frac{1}{2}$ *or* $\frac{1}{2} - \frac{\sqrt{3}}{4}$ b) $\frac{-\sqrt{3}}{\sqrt{2}}$ *or* $\frac{-\sqrt{6}}{2}$
36. 9.4 at 32°
37. $|\boldsymbol{H}| = 9.8;\ |\boldsymbol{V}| = 6.9; \boldsymbol{v} = 9.8\boldsymbol{i} + 6.9\boldsymbol{j}$
38. $\langle 2, -5\rangle$
39. 13
40. -8
41. 4
42. (A) Neither (B) Parallel (C) Orthogonal
43. $|\boldsymbol{u} + \boldsymbol{v}| = 23.3;\ \theta = 15.9°$
44. $\boldsymbol{u} = \langle\frac{-8}{17}, \frac{15}{17}\rangle$
45. (A) $\boldsymbol{v} = -5\boldsymbol{i} + 7\boldsymbol{j}$ (B) $\boldsymbol{v} = -3\boldsymbol{j}$ (C) $\boldsymbol{v} = -4\boldsymbol{i} - \boldsymbol{j}$
46. (A) Orthogonal (B) Not Orthogonal
47. (A) $\langle 2, -3\rangle$ (B) $\langle 6,3\rangle$ (C) $\langle 16,6\rangle$ (D) $\langle 15,8\rangle$
48. (A) 5**i** – 4**j** (B) **i** + 2**j** (C) 5**i** + 3**j** (D) 5**j**

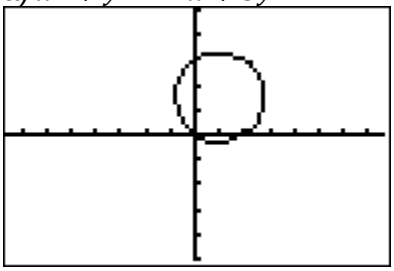
49.



50.



51. a) $(0, -3)$ b) $\left(\frac{10}{\sqrt{2}}, \frac{10}{\sqrt{2}}\right)$
52. a) $\left(-12, \frac{-\pi}{2}\right)$ b) $(-16, \pi)$
53. a) $r = 5sin\theta$ b) $r = \frac{8}{sin\theta}$
54. a) $x^2 + y^2 = 2x + 3y$



b) $x^2 + y^2 = 3x$