## Sum Identities:

$$
\begin{aligned}
& \cos (x+y)=\cos x \cos y-\sin x \sin y \\
& \sin (x+y)=\sin x \cos y+\cos x \sin y \\
& \tan (x+y)=\frac{\tan x+\tan y}{1-\tan x \tan y}
\end{aligned}
$$

## Difference Identities:

$$
\begin{aligned}
& \cos (x-y)=\cos x \cos y+\sin x \sin y \\
& \sin (x-y)=\sin x \cos y-\cos x \sin y \\
& \tan (x-y)=\frac{\tan x-\tan y}{1+\tan x \tan y}
\end{aligned}
$$

## Cofunction Identities:

$$
\begin{array}{ll}
\sin \left(\frac{\pi}{2}-x\right)=\cos x & \cos \left(\frac{\pi}{2}-x\right)=\sin x \\
\tan \left(\frac{\pi}{2}-x\right)=\cot x & \cot \left(\frac{\pi}{2}-x\right)=\tan x \\
\sec \left(\frac{\pi}{2}-x\right)=\csc x & \csc \left(\frac{\pi}{2}-x\right)=\sec x
\end{array}
$$

## Double-Angle Identities:

$$
\begin{aligned}
& \sin (2 x)=2 \sin x \cos x \\
& \cos (2 x)=\left\{\begin{array}{l}
\cos ^{2} x-\sin ^{2} x \\
1-2 \sin ^{2} x \\
2 \cos ^{2} x-1
\end{array} \quad \tan (2 x)=\left\{\begin{array}{l}
\frac{2 \tan x}{1-\tan ^{2} x} \\
\frac{2 \cot x}{\cot ^{2} x-1} \\
\frac{2}{\cot x-\tan x}
\end{array}\right.\right.
\end{aligned}
$$

## Half-Angle Identities:

$$
\begin{aligned}
& \sin \frac{x}{2}= \pm \sqrt{\frac{1-\cos x}{2}} \quad \cos \frac{x}{2}= \pm \sqrt{\frac{1+\cos x}{2}} \\
& \tan \frac{x}{2}= \pm \sqrt{\frac{1-\cos x}{1+\cos x}}=\frac{\sin x}{1+\cos x}=\frac{1-\cos x}{\sin x}
\end{aligned}
$$

## Product-Sum Identities:

$$
\begin{aligned}
& \sin x \cos y=\frac{1}{2}[\sin (x+y)+\sin (x-y)] \\
& \cos x \sin y=\frac{1}{2}[\sin (x+y)-\sin (x-y)] \\
& \sin x \sin y=\frac{1}{2}[\cos (x-y)-\cos (x+y)] \\
& \cos x \cos y=\frac{1}{2}[\cos (x+y)+\cos (x-y)]
\end{aligned}
$$

## Sum-Product Identities:

$$
\begin{aligned}
& \sin x+\sin y=2 \sin \frac{x+y}{2} \cos \frac{x-y}{2} \\
& \sin x-\sin y=2 \cos \frac{x+y}{2} \sin \frac{x-y}{2} \\
& \cos x+\cos y=2 \cos \frac{x+y}{2} \cos \frac{x-y}{2} \\
& \cos x-\cos y=-2 \sin \frac{x+y}{2} \sin \frac{x-y}{2}
\end{aligned}
$$

