

8.5 Complex Numbers

Imaginary unit $\sqrt{-1} = i$

also $i^2 = -1$

Complex number: A number written in the form $a + bi$ where a is a real number and b is the imaginary part.

Simplify the following using the imaginary number i when necessary.

a. $\sqrt{-4}$

b. $\sqrt{-50}$

c. $-\sqrt{25}$

For each complex number, name the real part and the imaginary part.

a. $7 + 2i$

b. $-8i$

Add or subtract the following complex numbers.

a. $(7 + 4i) + (6 + 5i)$

b. $(2 - 8i) - (6 + 3i)$

Multiply the following complex numbers.

a. $5(2 + 6i)$

c. $(4 + 7i)(2 - 9i)$

b. $3i(6 + 4i)$

Complex conjugates $a + bi$ and $a - bi$ are conjugates of one another.

Multiply the following complex numbers by their conjugates.

a. $3 + 5i$

b. $4 - 6i$

c. $-2i$

Divide the following. Put all answers in the standard form of a complex number ($a + bi$).

a. $\frac{9 + 21i}{3}$

b. $\frac{2 - 5i}{3 + 2i}$

c. $\frac{8 + 5i}{3i}$

Solve the following equations. Give answers in the standard form of a complex number.

a. $m^2 = -16$

b. $x^2 + 3x = -20$

Solve the following equations. Give answers in the standard form of a complex number.

c. $t^3 - 6t^2 + 10t = 0$

7.1 Rational Functions

Rational functions are functions that are fractions made up of polynomials.

Try graphing $f(x) = \frac{3x+1}{2x-6}$

What is the Domain?

What is the Range?

Find the domain of the following rational functions. Determine if the excluded values represent where a vertical asymptote appears in the graph.

a. $f(x) = \frac{x-7}{x+12}$

Find the domain of the following rational functions. Determine if the excluded values represent where a vertical asymptote appears in the graph.

b.
$$g(x) = \frac{x + 8}{x^2 + 6x - 16}$$

7.2 Simplifying Rational Expressions

To simplify rational expressions

1. Factor the numerator and denominator
2. Divide out any common factors
3. Leave in factored form

Simplify:

a. $\frac{40x^3}{12x}$

b. $\frac{(a+5)(a+7)}{(a-2)(a+5)}$

Simplify the following rational expressions.

a.
$$\frac{h^2 + 6h - 16}{h^2 - 4h + 4}$$

b.
$$\frac{(a - 2) + 8}{(a - 2)(a - 5)}$$

c.
$$\frac{12x^2 - 19x - 21}{15x^3 + 25x^2 - 140x}$$

Simplify the following rational expressions.

Special case – Sometimes we need to factor out a negative 1.

a. $\frac{3(x-4)}{5(4-x)}$