MW 8:30

Solving Quadratic Equations by Factoring

Names of Group Members:

Zero-Product Property: If $ab = 0$, then either $a = 0$ *or* $b = 0$, or both. (When two or more numbers are multiplied together and the result is zero, then one of them must equal zero.

Steps to Solve:

1. The equation must be in standard form $(ax^2 + bx + c = 0)$ and equal to zero. If it is not equal to zero, then you cannot use the zero-product property.
2. Factor the trinomial into 2 binomials.
3. Set each binomial equal to zero and solve for the variable.
4. You can verify these solutions in the original equation.

Examples:

1. $x^2 + 3x - 10 = 0$

   $(x+5)(x-2) = 0$

   $\boxed{x=-5}$ $\boxed{x=2}$

   $(-5)^2 + 3(-5) - 10 = 0$ $\quad$ $2^2 + 3(2) - 10 = 0$

   $0 = 0$ ✓ $\quad$ $4 + 6 - 10 = 0$ $\quad$ $0 = 0$ ✓

2. $10x^2 + 11x = 6$

   $10x^2 + 11x - 6 = 0$

   $(5x-2)(2x+3)$

   $5x - 2 = 0$ $\quad$ $\frac{5x}{5} = \frac{2}{5}$ $\quad$ $\boxed{x = 2/5}$

   $2x + 3 = 0$ $\quad$ $\frac{2x}{2} = \frac{-3}{2}$ $\quad$ $\boxed{x = -3/2}$

   $10(.4)^2 + 11(.4) = 6$ $\quad$ $10(.16) + 4.4 = 6$ $\quad$ $1.6 + 4.4 = 6$ $\quad$ $6 = 6$ ✓

   $10(-1.5)^2 + 11(-1.5) = 6$ $\quad$ $22.5 - 16.5 = 6$ $\quad$ $6 = 6$ ✓

3. $3x^2 + 7x = 6$

   $3x^2 + 7x - 6 = 0$

   $(3x-2)(1x+3)$ $\quad$ $\boxed{x = -3}$

   $3x - 2 = 0$ $\quad$ $\frac{3x}{3} = \frac{2}{3}$ $\quad$ $\boxed{x = 2/3}$

   $3(-3)^2 + 7(-3) = 6$ $\quad$ $3(9) - 21 = 6$ $\quad$ $27 - 21 = 6$ $\quad$ $6 = 6$ ✓

   $3(\frac{2}{3})^2 + 7(\frac{2}{3}) = 6$ ✓

4. The height above ground of a ball thrown at 64 feet per second from the top of an 80-foot-high building is modeled by $S = 80 + 64t - 16t^2$ feet, where t is the number of seconds after the ball is thrown. How long will the ball be in the air?

   $-16t^2 + 64t + 80$

   $-16(t^2 - 4t - 5)$

   $-16(t-5)(t+1)$

   $\boxed{t = 5}$ $\cancel{\boxed{t = -1}}$

   $-16(5)^2 + 64(5) + 80$ $\quad$ $-400 + 320 + 80 = 0$ $\quad$ $0 = 0$ ✓

   $-16(-1)^2 + 64(-1) + 80$ $\quad$ $-16 - 64 + 80 = 0$ $\quad$ $0 = 0$ ✓

Solving Quadratic Equations by Graphical & Numerical Methods

2-13-13

Names of Group Members:

Graphical: where the graph intercepts the x-axis is the solution(s) to the quadratic equation.

Steps to Solve:

1. The equation must be in standard form $(ax^2 + bx + c = y)$.
2. Set y_1 = the equation, then graph.
3. Use 2^{nd} Trace option 2:zero, identify left bound, right bound, and guess for calculator to identify zero.
4. You can verify these solutions in the original equation.

Examples:

1. $2x^2 + x - 6 = 0$

   $x = 1.5$ $\quad x = -2$

   CHECK:

   $2(1.5)^2 + (1.5) - 6 = 0$ ✓

   $2(-2)^2 + (-2) - 6 = 0$ ✓

2. $6x^2 + 5x - 6 = 0$

   $x = -1.5$ $\quad x = \frac{2}{3}$

   CHECK

   $6(-1.5)^2 + 5(-1.5) - 6 = 0$ ✓

   $6(2/3)^2 + 5(2/3) - 6 - 0 = 0$ ✓

3. $10x^2 = 22x - 4$

   $10x^2 - 22x + 4 = 0$

   $x = 0.2$ $\quad x = 2$

   CHECK

   $10(0.2)^2 - 22(0.2) + 4 = 0$ ✓

   $10(2)^2 - 22(2) + 4 = 0$ ✓

4. $x^2 + 4x = -8$

   $+8 \quad +8$

   $x^2 + 4x + 8 = 0$

   no real solution.

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Names of Group Members:

Graphical: where the graph intercepts the x-axis is the solution(s) to the quadratic equation.

Steps to Solve:

1. The equation must be in standard form $(ax^2 + bx + c = y)$.
2. Set $y_1 =$ the equation, then graph.
3. Use 2nd Trace option 2:zero, identify left bound, right bound, and guess for calculator to identify zero.
4. You can verify these solutions in the original equation.

Examples:

1. $2x^2 + x - 6 = 0$

   $x = 1.5$ $\quad$ $x = -2$

   CHECK:

   $2(1.5)^2 + (1.5) - 6 = 0$ ✓

   $2(-2)^2 + (-2) - 6 = 0$ ✓

2. $6x^2 + 5x - 6 = 0$

   $x = -1.5$ $\quad$ $x = \frac{2}{3}$

   CHECK

   $6(-1.5)^2 + 5(-1.5) - 6 = 0$ ✓

   $6(2/3)^2 + 5(2/3) - 6 = 0 = 0$ ✓

3. $10x^2 = 22x - 4$

   $10x^2 - 22x + 4 = 0$

   $x = 0.2$ $\quad$ $x = 2$

   CHECK

   $10(0.2)^2 - 22(0.2) + 4 = 0$ ✓

   $10(2)^2 - 22(2) + 4 = 0$ ✓

4. $x^2 + 4x = -8$

   $+8 \quad +8$

   $x^2 + 4x + 8 = 0$ $\quad$ no real solution

Solving Quadratic Equations by Square Root Method

Names of Group Members:

Square Root Property: If $x^2 = a, then\ x = \sqrt{a}\ or\ x = -\sqrt{a}$, so the solutions of the quadratic equation of the form $x^2 = a$ are given by $x = \pm\sqrt{a}$.

Steps to Solve:

1. The equation must not have a bx term.
2. Isolate the squared term.
3. Apply the square root property, by taking the square root of both sides.
4. If there is a variable expression, then solve for the variable.
5. You can verify these solutions in the original equation.

Examples:

1. $x^2 = 20$

$\sqrt{x^2} = \sqrt{20}$

$x = \pm\sqrt{20} = \sqrt{4}\sqrt{5}$

$x = \pm 2\sqrt{5}$

2. $5x^2 - 25 = 0$

$+25\ +25$

$\frac{5x^2}{5} = \frac{25}{5}$

$\sqrt{x^2} = \sqrt{5}$

$x = \pm\sqrt{5}$

3. $\sqrt{(x-6)^2} = \sqrt{18}$

$x - 6 = \pm\sqrt{18}$

$+6\ +6$

$x = 6 \pm 3\sqrt{2}$

4. $(2x+1)^2 + 7 = 0$

$-7\ -7$

$\sqrt{(2x+1)^2} = \sqrt{-7}$

$2x + 1 = \pm i\sqrt{7}$

$-1\ -1$

$\frac{2x}{2} = \frac{-1 \pm i\sqrt{7}}{2}$

$x = \frac{-1 \pm i\sqrt{7}}{2}$ or $-\frac{1}{2} \pm \frac{i\sqrt{7}}{2}$

Solving Quadratic Equations by Completing the Square

Names of Group Members:

Completing the Square: when you can't factor the trinomial then your split up the trinomial and can use a version of the square root method.

Steps to Solve:

1. The equation must have the variables on one side of the equal sign and the constant on the other side ($x^2 + bx = d$).
2. First you must determine what the "c" value should be in order to create a perfect square trinomial: $c = \left(\frac{b}{2}\right)^2$.
3. Then add $\left(\frac{b}{2}\right)^2$ to both sides.
4. Rewrite the perfect square trinomial in the form of a binomial squared $\left(x + \frac{b}{2}\right)^2$ and simplify the constants on the right hand side.
5. Now use the square root method to solve for x.
6. You can verify these solutions in the original equation.

Examples:

1. $x^2 + 4x - 9 = 0$

   $\left(\frac{4}{2}\right)^2$

   $x^2 + 4x + \underline{4} = 9 + \underline{4}$

   $\sqrt{(x+2)^2} = \sqrt{13}$

   $x + 2 = \pm\sqrt{13}$

   $x = -2 \pm \sqrt{13}$

2. $x^2 - 12x = -17$

   $\left(\frac{-12}{2}\right)^2$

   $x^2 - 12x + \underline{36} = -17 + \underline{36}$

   $\sqrt{(x-6)^2} = \sqrt{19}$

   $x - 6 = \pm\sqrt{19}$

   $x = 6 \pm \sqrt{19}$

3. $x^2 - 6x + 1 = 0$

   $\left(\frac{6}{2}\right)^2 = 9$

   $x^2 - 6x + \underline{9} = -1 + \underline{9}$

   $(x-3)^2 = \sqrt{8}$

   $x - 3 = \pm\sqrt{8}$

   $x = 3 \pm \sqrt{8}$ $\quad \boxed{x = 3 \pm 2\sqrt{2}}$

4. $x^2 - 3x + 3 = 0$

   $\left(\frac{3}{2}\right)^2$

   $x^2 - 3x + \underline{9/4} = -3 + \underline{9/4}$

   $\sqrt{\left(x - \frac{3}{2}\right)^2} = \sqrt{-3/4}$

   $x - \frac{3}{2} = \frac{\pm i\sqrt{3}}{2}$

   $x = 3/2 \pm \frac{i\sqrt{3}}{2}$

$$\left(\frac{b}{2}\right)^2$$

$$x^2 - 12x = -17$$

$$x^2 - 12x + \underline{36} = -17 + \underline{36}$$

$$\sqrt{(x-6)^2} = \sqrt{19}$$

$$x - 6 = \pm\sqrt{19}$$

$$x = 6 \pm \sqrt{19}$$

Solving Quadratic Equations by Quadratic Formula

Names of Group Members:

Quadratic Formula: solutions of the quadratic equation of the form $ax^2 + bc + c = 0$ are given by $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$.

Steps to Solve:

1. The equation must be in standard form $(ax^2 + bx + c = 0)$.
2. Replace each letter with its appropriate coefficients from the quadratic equation.
3. Simplify the formula.
4. You can verify these solutions in the original equation.

Examples:

1. $x^2 + 3x - 10 = 0$

$X = \frac{-3 \pm \sqrt{3^2 - 4(1)(-10)}}{2} = \frac{-3 \pm 7}{2}$

$X_1 = \frac{-10}{2} = -5 \qquad X_2 = \frac{4}{2} = 2$

2. $3x^2 - 30x - 180 = 0$

$3(x^2 - 10x - 60) = \emptyset$

$X = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(-60)}}{2}$

$X = \frac{10 \pm \sqrt{340}}{2} \Rightarrow \frac{10}{2} \pm \frac{\sqrt{4}\sqrt{85}}{2}$

$X = 5 \pm \sqrt{85}$

3. $6 - 3x^2 + 4x = 0$

$-3x^2 + 4x + 6 = \emptyset$

$X = \frac{-4 \pm \sqrt{(4)^2 - 4(-3)(6)}}{2(-3)}$

$X = \frac{-4 \pm \sqrt{88}}{-6} \Rightarrow X = \frac{-4}{-6} \pm \frac{\sqrt{4}\sqrt{22}}{-6}$

$X_1 = \frac{2}{3} - \frac{\sqrt{22}}{3}; \quad X_2 = \frac{2}{3} + \frac{\sqrt{22}}{3}$

4. $3x^2 + 4x = -3$

$3x^2 + 4x + 3 = \emptyset$

$X = \frac{-4 \pm \sqrt{(4)^2 - 4(3)(3)}}{2 \cdot 3} = \frac{-4 \pm \sqrt{-20}}{6}$

$X = \frac{-4}{6} \pm \frac{\sqrt{-4}\sqrt{5}}{6} \Rightarrow \frac{-2}{3} \pm \frac{\sqrt{-1}\sqrt{4}\sqrt{5}}{6}$

$= \frac{-2}{3} \pm \frac{2i\sqrt{5}}{6}$

$= \frac{-2}{3} \pm \frac{i\sqrt{5}}{3}$