College Algebra
Final Exam Practice

I. Solve the system of equations using the method of your choice. Identify whether the system is consistent, inconsistent and dependent or independent.

1. $\begin{aligned} -5x + 7y &= -40 \\ -2x + 4y &= -16 \end{aligned}$

2. $\begin{aligned} 3x + y &= 10 \\ 6x + 2y &= 20 \end{aligned}$

II. Use the square root method to solve the equation. Leave answer in simplified radical form or simplified form.

3. $2z^2 - 72 = 0$

4. $(x - 5)^2 = 19$

III. Answer the following questions:
a) state the vertex of the function
b) identify any maxima and minima
c) identify the axis of symmetry
d) state the domain
e) state the range
f) state the intervals in which *f* is increasing and/or decreasing

5. $f(x) = -2x^2 - 16x - 26$

6. $f(x) = \frac{1}{2}x^2 - 2x + 5$

IV. Find the slope-intercept form for the line that satisfies the stated conditions.

7. Perpendicular to $y = \frac{4}{5}(x - 7) + 7$, passing through $(8, -1)$

V. Graph the solution.

8. $x + y \leq 2$

VI. For the following problems, solve by:
a) Assign variables to the unknowns
b) Write a system of equations
c) Solve the system, you must show some work
d) Write the solution in terms of the problem

9. Jim wants to plan a meal with 66 grams of carbohydrates and 1040 calories. If green beans have 7 grams of carbohydrates and 30 calories per half-cup serving and French fried shrimp have 9 grams of carbohydrates and 190 calories per three-ounce serving, how many servings of green beans and shrimp should he use?

10. Nadine sold two kinds of tickets to her class play. Student tickets cost $4.00 each, and adult tickets cost $6.50 each. If Nadine sold a total of 35 tickets for $182.50, how many student and adult tickets did she sell?

VII. Solve the following problems.

11. A manufacturer has total revenue given by the function $R = 280x$ and has total cost given by $C = 69x + 297{,}000$; where x is the number of units produced and sold. Find the number of units where the product breaks even.

12. The demand for a certain product is given by $p + 5q = 328$, and the supply is given by $p - 7q = 28$, where p is the price in dollars and q is the quantity demanded or supplied at price p. Find the price and quantity at which market equilibrium occurs.

13. John owns a hot dog stand. He has found that his profit is given by the equation $P = -x^2 + 50x + 74$, where x is the number of hot dogs sold.
a) How many hot dogs must he sell to earn the most (max) profit?
b) What is that profit?

14. Given the following revenue and cost functions. (Recall that profit equals revenue minus cost.) $R(x) = 78x - 2x^2; C(x) = 22x + 106$, where x is the number of units.
a) Find P(x) and simplify it.
b) Graphically find the number of units for maximum profit and what that profit is.

15. The number of high school students who have ever used marijuana for the years 1990 through 2006 is given by $y = -0.1940x^2 + 4.0142x + 27.861$, where *x* is the number of years from 1990. Use the model to estimate the years when the percent who ever used marijuana is greater than 42.4%.

16. The percent of US population that is foreign born can be modeled by the function$P(x) = 0.003x^2 - 0.42x + 19.8$, where x is the number of years after 1900.
a) What is the percent of US population that is foreign born in 1997?
b) What year(s) was the percent of US population foreign born 7.8?

VIII. Graph the system of inequalities. Shade the solution area AND state the intersection points.

17. $2x + 3y > 6$
$x - y \leq 3$

IX. Use the function, to answer the following:

18. $f(x) = x^2 - 9x + 18$
a) Use the discriminant to determine the number and type of solutions of the quadratic equation.
b) Determine the concavity of $f(x)$.
c) Find the vertex of $f(x)$.
d) Find the axis of symmetry.
e) Find the x-intercept(s), and write the intercept(s) as ordered pair(s).
f) Find the y-intercept.
g) State the domain of the function.
h) State the range of the function.
i) Sketch a graph of the parabola using the information obtained in the parts above.

X. Write the vertex form of the parabola that satisfies the given conditions.

19. $Vertex = (-3,4)\ and\ passing\ through\ the\ point\ (2,9)$
20. $Vertex = (-1,-5)\ and\ passing\ through\ the\ point\ (-4,-14)$

XI. Solve the inequality by using/showing a number line. Write the solution in interval notation.

21. $x^2 - 6x \geq -8$

XII. Solve the problem.

22. Assume that the elevation E, in feet, of a sag in a proposed route is given by $E(x) = 0.000035x^2 - 0.3x + 1400$, where x represents the horizontal distance in feet along the proposed route and $0 \le x \le 5000$. For what **x-values** is the elevation 1250 feet or more? Round your answer to the nearest foot. Hint: write a quadratic inequality, then solve it.

23. The supply function for a product is $p(x) = \frac{1}{3}x^2 + 30, x \ge 0$, where x is the number of thousands of units a manufacturer will supply if the price is p(x) dollars.
a) Find the inverse of this function.
b) For this problem, what does the inverse compute?
c) **Using the inverse function**, how many units are supplied if the price is $462?

XIII. Find the inverse of the function.

24. $f(x) = \frac{x}{7} - 3$

25. $f(x) = 7x^3 - 6$

XIV. Solve the equation, algebraically. Round to three decimal places.

26. $e^{0.386x} = 21$

27. $4^{(x-3)} = 15$

28. $e^{lnx} = 11$

29. $lnx - \ln(x-4) = ln2$

30. $\log_5(x+9) + \log_5(x-9) = 1$

XV. Sketch a graph of the function.

31. $$f(x) = \begin{cases} x+3 & if \quad x > 4 \\ -5 & if \quad -3 < x \le 4 \\ 1-x & if \quad x \le -3 \end{cases}$$

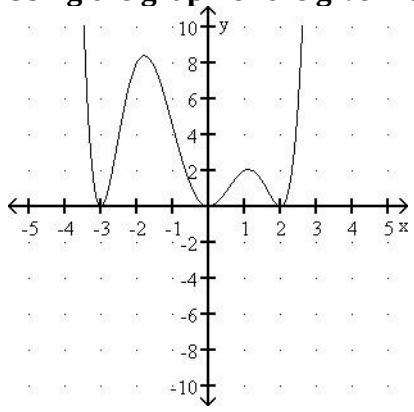
32. Using the piecewise function from # 31 evaluate the following.
a) $f(2) =$ b) $f(11) =$ c) $f(-7) =$

33. As part of a water conversation program, the public water company in Arid, New Mexico establishes a three tier system for monthly bills based on use of hundred cubic feet of water usage. Where x is the number of hundred cubic feet a household uses.

$$f(x) = \begin{cases} 50 + 2x & if \quad x \le 50 \\ 50 + 3.5x & if \quad 50 < x \le 110 \\ 50 + 5.75x & if \quad x > 110 \end{cases}$$

Use f to find the monthly bill for usage of hundred cubic feet is:
a) 132 b) 37 c) 68

XVI. Using the graph of the given function, identify the following.



34. a) $f(-2) =$ b) $f(1) =$ c) Find x-values where $f(x) = 0$

XVII. Write the equation of the graph after the indicated transformation(s).

35. Starting with the basic absolute value function, the graph is reflected over the x-axis, shifted horizontally left by 3 units, shifted vertically down by 8 units and compressed by a factor of 0.6.

XVIII. Use the given function to complete the following. Be able to use any of the basic functions.

a) Sketch the graph b) Identify the domain c) Identify the range.

36. $f(x) = -3 + \sqrt{x+2}$

37. $f(x) = \frac{1}{x}$

38. $f(x) = \frac{1}{x} + 4$

XIX. Find the requested composition of functions.

39. Given $f(x) = 4x^2 + 2x + 5 \; and \; g(x) = 2x - 3$,

a) Find $(g \circ f)(x)$ b) Find $(f \circ g)(x)$

XX. For the pair of functions, perform the indicated operations.

40. $f(x) = 5x^2 - 8x \; and \; g(x) = x^2 - 3x - 40$

a) Find $(g + f)(x)$

b) Find $(g - f)(x)$

c) Find $(f - g)(x)$

d) Find $(fg)(x)$

e) Find $\left(\frac{f}{g}\right)(x)$ and its domain.

f) Find $\left(\frac{g}{f}\right)(x)$ and its domain.

XXI. Solve the problem.

41. One method to determine the time since an animal died is to estimate the percentage of carbon-14 remaining in its bones. The percent P in decimal form of carbon-14 remaining x years is given by $P(x) = e^{-0.000121x}$. Approximate (to the nearest whole year) the age of a fossil if there is 41% of carbon-14 remaining. (hint P(x)=0.41)

42. Suppose the number of Quickie hamburgers (in millions) served yearly from 1987 to 2000 can be modeled by $f(x) = 39.8e^{0.12x}$. In this formula x=0 corresponds to 1987 and x=13 corresponds to 2000.

a) Approximate how many million (rounded to the nearest million) Quickie hamburgers will be sold in 2000.

b) In what year (to the nearest year) did Quickie serve 117.2 million hamburgers?

43. Under ideal conditions, a population of rabbits has an exponential growth rate of 11.5% per day. Consider an initial population of 900 rabbits.

a) Find the exponential growth function. (Do not use e in the function)

b) How many rabbits will you have in 5.5 days? (to the nearest rabbit)

c) How many days will it take to have 7621 rabbits?

44. A colony of bacteria decreases in size every 6 hours by half of its previous amount when an antibiotic is given. The original amount was estimated at 15000.

a) Find the exponential decay function. (Do not use e in the function)

b) How many bacteria are left after 3 hours?

c) How many hours will it take till the colony is down to 1052 bacteria? (to the nearest hour)

XXII. Use the change of base formula to approximate to four decimal places.

45. $\log_{5.2} 178$ 46. $\log_3 0.435$

XXIII. Use the equation of the polynomial function to

a) State the degree
b) State the leading coefficient
c) Find the y-intercept(s) of $f(x)$.
d) Describe the end behavior of the graph of the function

47. $f(x) = -20x^3 + 6x - 7$ 48. $f(x) = 12x^4 - 3x^3 + 9$
49. $f(x) = -8(x + 2)(x^5 - 2)$

XXIV. Solve the problem.

50. A ball is thrown upward at 96 feet per second from the top of a building that is 100 feet high. Using the function, $h(t) = -16t^2 + v_0t + h_0$, with the appropriate substitutions, answer the following questions. If necessary round to the nearest hundredths.
a) Write the appropriate function.
b) Determine the time it takes the ball to attain its maximum height.
c) Determine the maximum height the ball attains.
d) Determine the time it takes the ball to hit the ground.

51. A rectangular piece of cardboard measuring 17 inches by 43 inches is to be made into a box with an open top by cutting equal size squares from each corner and folding up the sides. Let x represent the length of a side of each such square. For what value of x will the volume be a maximum? If necessary, round to 2 decimal places.

52. The Mad Hatter is ordering cups from Teacups, Limited, for his tea party. The Teacups, Limited catalog prices cups according to the number of cups ordered. For orders of 20 or fewer cups, the price is $1.40 per cup plus $12 shipping and handling on the order. For orders of more than 20 cups, the price is $1.10 per cup plus $15 shipping and handling.
a) Write a piecewise-defined function that represents Teacups, Limited cup prices.
b) How much will The Mad Hatter owe if he orders 15 teacups?
c) How many teacups did The Mad Hatter order if the cost was $44.70?

XXV. Find a) the domain b) any removable discontinuities (holes) c) horizontal asymptotes d) vertical asymptotes e) sketch a graph of the function.

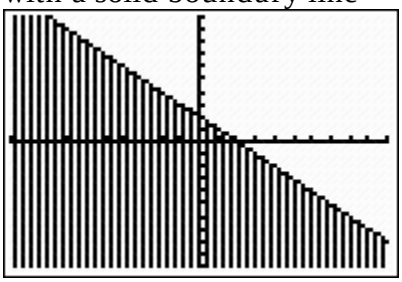
53. $f(x) = \frac{x^2+x-9}{x^2-9}$ 54. $f(x) = \frac{x^2-4}{x-2}$
55. $f(x) = \frac{x-3}{x+4}$

XXVI. Solve the polynomial equation.

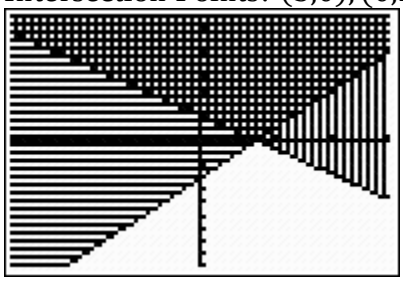
56. $(2x - 7)(x + 5)(x - 5) = 0$ 57. $x^3 - 5x^2 + 6x = 0$
58. $4x^3 - 24x^2 - x + 6 = 0$ 59. $x^4 - 6x^2 + 8 = 0$
60. $\frac{1}{7}x^3 + 49 = 0$

Answers:

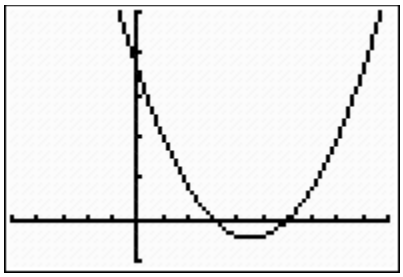
1. (8,0), consistent, independent
2. Infinitely many solutions, Dependent System
3. $z = \pm 6$
4. $x = 5 \pm \sqrt{19}$
5. a) $(-4,6)$ b) $Max = (-4,6)$ c) $x = -4$ d) $(-\infty, \infty)$ e) $(-\infty, 6]$
   f) $Increasing\ (-\infty, -4); decreasing\ (-4, \infty)$
6. a) (2,3) b) $Min = (2,3)$ c) $x = 2$ d) $(-\infty, \infty)$ e) $[3, \infty)$
   f) $Increasing\ (2, \infty); decreasing\ (-\infty, 2)$
7. $y = \frac{-5}{4}x + 9$
8. with a solid boundary line



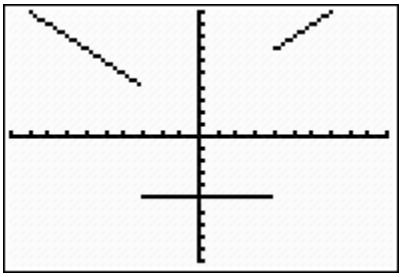
9. a) x = # of servings of green beans, y = # of servings of shrimp
   b) $\begin{matrix} 7x + 9y = 66 \\ 30x + 190y = 1040 \end{matrix}$
   c) (3,5)
   d) You need 3 servings of green beans and 5 servings of shrimp.
10. a) x = # of student tickets, y = # of adult tickets
    b) $\begin{matrix} x + y = 35 \\ 4x + 6.5y = 182.50 \end{matrix}$
    c) (18,17)
    d) She sold 18 student tickets and 17 adult tickets.
11. For break-even point we need 1408 units.
12. For market equilibrium we need 25 units at a price of $203.
13. a) John needs to sell 25 hot dogs for a maximum profit.
    b) The maximum profit is $699.
14. a) $P(x) = -2x^2 + 56x - 106$ b) We need 14 units for a maximum profit of $286.
15. From 1995 through 2006 the percent who ever used marijuana is greater than 42.4%.
16. a) In 1997, the percent of US population that is foreign born was 7.287%.
    b) In the years 1940 and 2000, the % of US population that is foreign born was 7.8%.
17. Intersection Points: (3,0), (0,2)



18. a) $b^2 - 4ac = 9 > 0$, so 2 real solutions
    b) concave up
    c) $(4.5, -2.25)$
    d) $x = 4.5$
    e) $(6,0), (3,0)$
    f) $(0,18)$
    g) $(-\infty, \infty)$
    h) $[-2.25, \infty)$
    i)

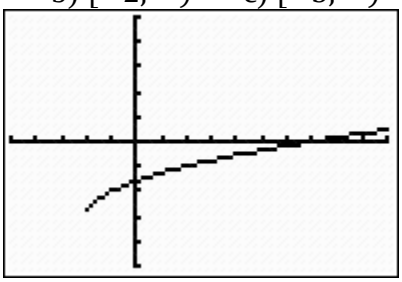


19. $y = \frac{1}{5}(x+3)^2 + 4$
20. $y = -(x+1)^2 - 5$
21. $(-\infty,2] \cup [4,\infty)$
22. We need a horizontal distance of less than 533 feet.
23. a) $f^{-1}(x) = \sqrt{3x - 90}$ b) Given the price in dollars, f^{-1} computes the number of units (in thousands) c) For the price of \$462, we will have 36 thousand units.
24. $f^{-1}(x) = 7x + 21$
25. $f^{-1}(x) = \sqrt[3]{\frac{x+6}{7}}$
26. $x \approx 7.887$
27. $x \approx 4.953$
28. $x = 11$
29. $x = 8$
30. $x = \sqrt{86} \approx 9.274$
31.

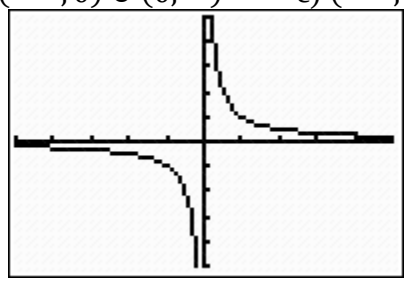


32. a) $f(2) = -5$ b) $f(11) = 14$ c) $f(-7) = 8$
33. a) Your monthly bill will be \$809 for 132 cubic feet used. b) Your monthly bill will be \$124 for 37 cubic feet used. c) Your monthly bill will be \$288 for 68 cubic feet used.
34. a) 8 b) 2 c) $x = -3, x = 0, x = 2$
35. $f(x) = -0.6|x+3| - 8$

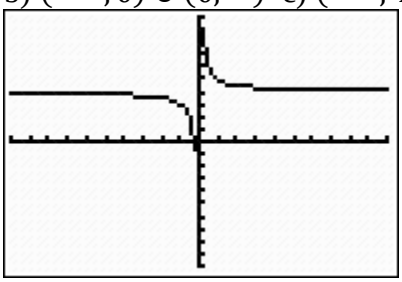
36. b) $[-2, \infty)$ c) $[-3, \infty)$



37. b) $(-\infty, 0) \cup (0, \infty)$ c) $(-\infty, 0) \cup (0, \infty)$

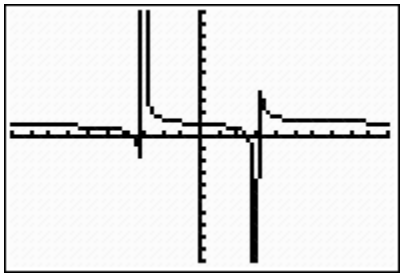


38. b) $(-\infty, 0) \cup (0, \infty)$ c) $(-\infty, 4) \cup (4, \infty)$

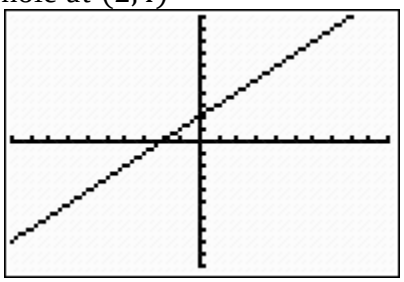


39. a) $(g \circ f)(x) = 8x^2 + 4x + 7$ b) $(f \circ g)(x) = 16x^2 - 44x + 35$
40. a) $(g + f)(x) = 6x^2 - 11x - 40$ b) $(g - f)(x) = -4x^2 + 5x - 40$
    c) $(f - g)(x) = 4x^2 - 5x + 40$ d) $(fg)(x) = 5x^4 - 23x^3 - 176x^2 + 320x$
    e) $\left(\frac{f}{g}\right)(x) = \frac{5x^2 - 8x}{x^2 - 3x - 40}, domain: x \in \mathbb{R}, x \neq -5, 8$
    f) $\left(\frac{g}{f}\right)(x) = \frac{x^2 - 3x - 40}{5x^2 - 8x}, domain: x \in \mathbb{R}, x \neq 0, \frac{8}{5}$
41. The fossil is 7,369 years old.
42. a) In 2000 Quickie sold 189 million hamburgers.
    b) In 1996 Quickie sold 117.2 million hamburgers.
43. a) $P(t) = 900(1.115^t)$
    b) In 5.5 days you will have 1638 rabbits.
    c) It will take 19.6 days to have 7621 rabbits.
44. a) $P(t) = 15000\left(0.5^{t/6}\right)$
    b) In 3 hours you will have 10,607 bacteria.
    c) It will take 23 hours to reach 1052 bacteria.
45. $\frac{\log 178}{\log 5.2} \approx 3.1430$
46. $\frac{\log 0.435}{\log 3} \approx -0.7577$
47. a) 3 b) -20 c) $f(0) = -7$ d) Left end rises and right end falls
48. a) 4 b) 12 c) $f(0) = 9$ d) Both ends rise
49. a) 6 b) -8 c) $f(0) = 32$ d) Both ends fall
50. a) $h(t) = -16t^2 + 96t + 100$ b) 3 seconds c) 244 feet d) 6.91 seconds
51. 3.75 inches
52. a) $f(x) = \begin{cases} 1.40x + 12, if\ x \leq 20 \\ 1.10x + 15, if\ x > 20 \end{cases}$ b) \$33.00 c) 27 cups

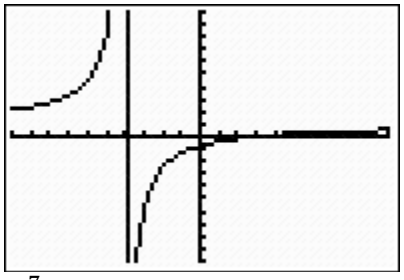
53. a) $(-\infty,-3)\cup(-3,3)\cup(3,\infty)$ b) None c) $y=1$ d) $x=\pm 3$
e)



54. a) $(-\infty,2)\cup(2,\infty)$ b) hole at $(2,4)$ c) none d) none
e) hole at $(2,4)$



55. a) $(-\infty,-4)\cup(-4,\infty)$ b) None c) $y=1$ d) $x=-4$
e)



56. $x=\frac{7}{2},-5,\ 5$
57. $x=0,2,3$
58. $x=\frac{1}{2},\frac{-1}{2},6$
59. $x=\pm 2,\pm\sqrt{2}$
60. $x=-7$