**Partial Fractions Decomposition:**

Partial fraction is the decomposition of a rational expression into component fractions.

***Case I Distinct Roots* (Factors)*:***

Consider the proper rational function where the degree of is less than the degree of, and *a* ≠ *b.* The partial fraction expansion will be given by

 , **(1)**

 where A and B must be determined.

If we multiply (1) by, we obtain

. If we evaluate at, we obtain that evaluated at. This is the same as covering-up in and evaluating at in the part of the fraction that is visible; that is.

To find B, all we have to do is cover-up in and evaluate at .

This is what is known as the cover-up method.

**NOTE:** If degree of the numerator is greater than or equal to the degree of the denominator, long division must be performed first and then partial fraction expansion can be applied to the remainder by divisor part.

Example: Find the partial fraction decomposition of .

By the cover up method, evaluated at. Notice that we have cover-up the *x* in the denominator, so

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So, the partial fraction decomposition is

Note: If you solve this problem using the book’s method, you will need to solve a system of 3 equations.

***Case II Repeated Roots* (Factors)*:***

Consider the proper rational function , where is a repeated root. The partial fraction expansion will be given by

 , where A, B and C must be determined.

When you have repeated roots, you have to account for the original factor, and all the lower powers of that factor.

In this case, A and C can be determined using the cover-up methods (note that those are the factors in the original function).

Example: Find the partial fraction decomposition of .

 **(2)**

A and B can be found using the cover up method (same as the original factors)

By the cover up method, evaluated at, so

By the cover up method, evaluated at, so.

To find C and D, we have to solve a system of equations.

Since we know A and B, if we multiply **(2)** times, we obtain

 **(3)**

If we substitute any number other than the ones already used (say) in **(3),** we obtain

C+D = 0

If we substitute any other number (say) in **(3),** we obtain

C-2D = -3

If we solve the system, we get D=1 and C= -1

So, the partial fraction decomposition is

Note: If you solve this problem using the book’s method, you will need to solve a system of 4 equations.

***Case III Quadratic Roots:***

Consider the proper rational function , where

is any quadratic factor with real or imaginary solutions. The partial fraction expansion will be given by

 , where A, B and C must be determined.

In this case we can use the cover-up method in the linear term, and a systems of equations find.

Example: Find the partial fraction decomposition of

 **(4)**

By the cover up method, evaluated at, so.

Since we know by cover-up, if we multiply **(4)** times, we obtain **(5)**

At this point, you can substitute any number for *x.*

If we substitute in **(5)** we obtain

2B+C= 1

If we substitute in **(5)** we obtain

B-C = 1/2

If we solve the system, we get B=1/2 and C= 0

So, the partial fraction decomposition is

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Note: Skip this part. In some cases, this method can be more complicated than the one shown above. The same example can be solved by using cover-up in both terms.

 **(6)**

By the cover up method, evaluated at, so

By the cover up method, evaluated at, one of the two solutions of

So. If we rearrange and multiply the right hand side by the conjugate, we obtain

. If we equate the 2 sides of the equation, we obtain, and.

From these two equations, we get and.

The partial fraction decomposition of

.

Cover-up for this case can become complicated, so in many cases is better to use the systems of equations method.

Note: Applications of Partial Fraction Decomposition will be studied in Calculus II, and in Differential Equations.

Practice:

Expand into Partial Fractions

 Function Answer

 

 g(*x*) := 

  

 

 

 