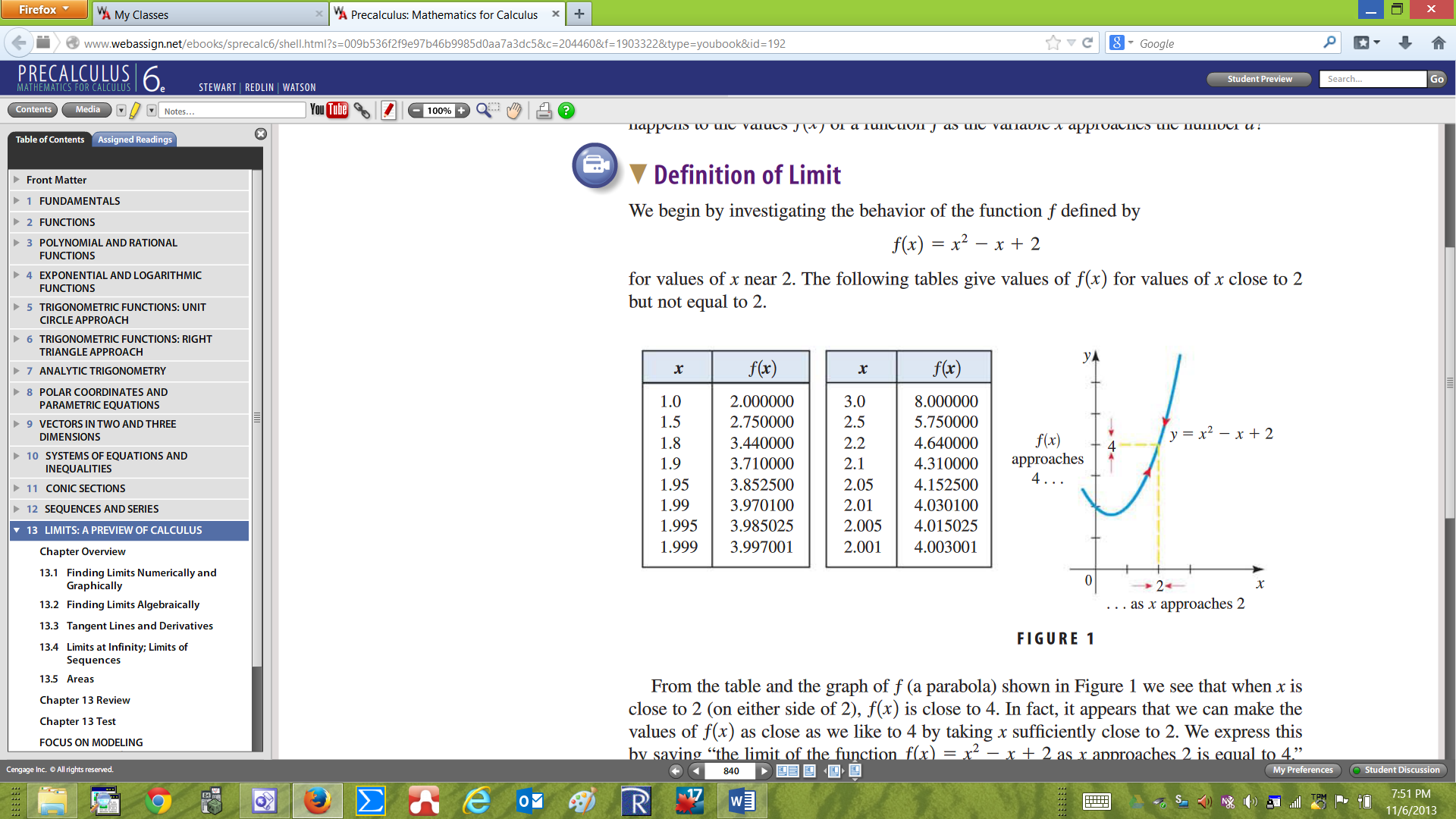
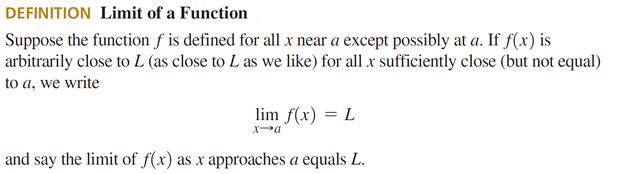
**The limit of a function (Basics)**

Consider the function for values of *x* near 2. The table below gives the values of when *x* is closed to 2, but not equal to 2. We can see that the closer x approaches 2 from either side, the closer approaches 4.



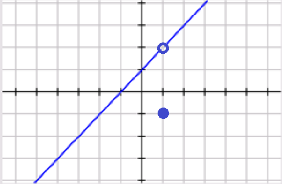
To express this situation we can write.

In general, the limit of the function tell us what value the function is “approaching to” when is approaching a number say in the domain of the function. The limit of the function does not refer to *f* (a) (the value the function takes at.)



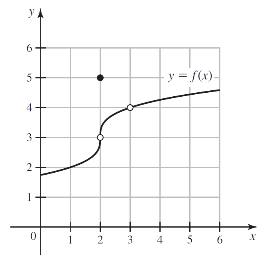
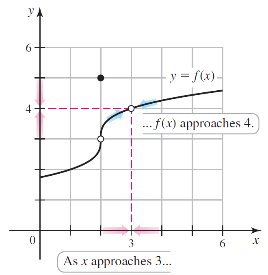
**Example1**: Consider the piecewise function. Find, and

Solution: This is a linear function with a discontinuity at. We can see that when x approaches 1, approaches 2, so, whereas.



From this example we can see that.

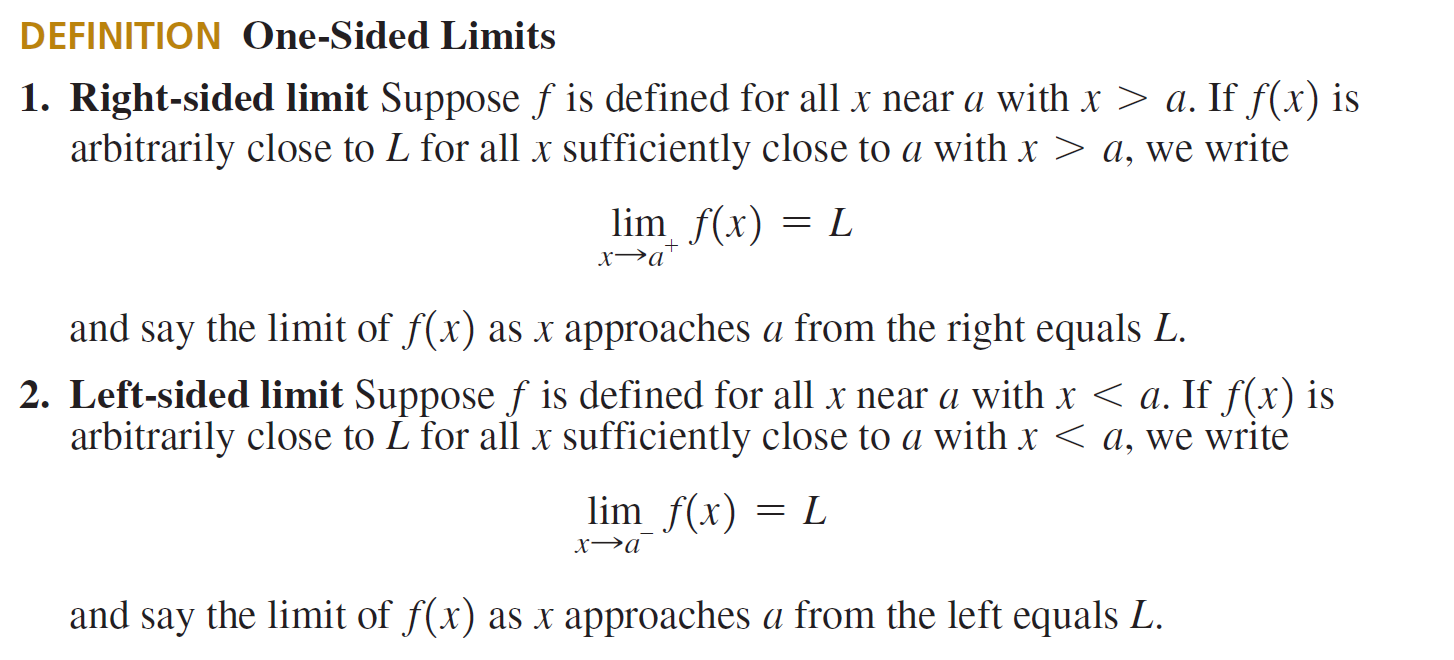
**Example 2**: Find, and in the figure below:

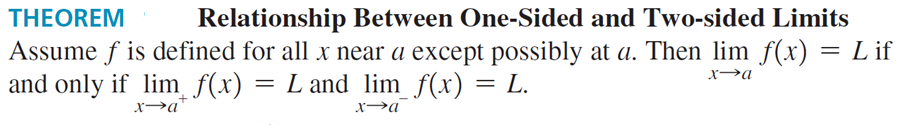
 

Solution: We can see that when x approaches 3, approaches 4, so. Since 3 is not in the domain of , is undefined.

From this example we can see that the limit exists at 3 even though is undefined.

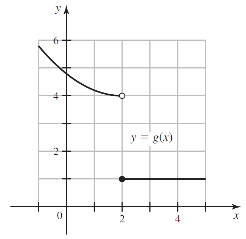
**One- Sided Limits**





The theorem tells us that for the limit to exist, the limit from the right has to equal the limit from the left.

**Example 3**: **A Function with a Jump** Findand in the figure below:

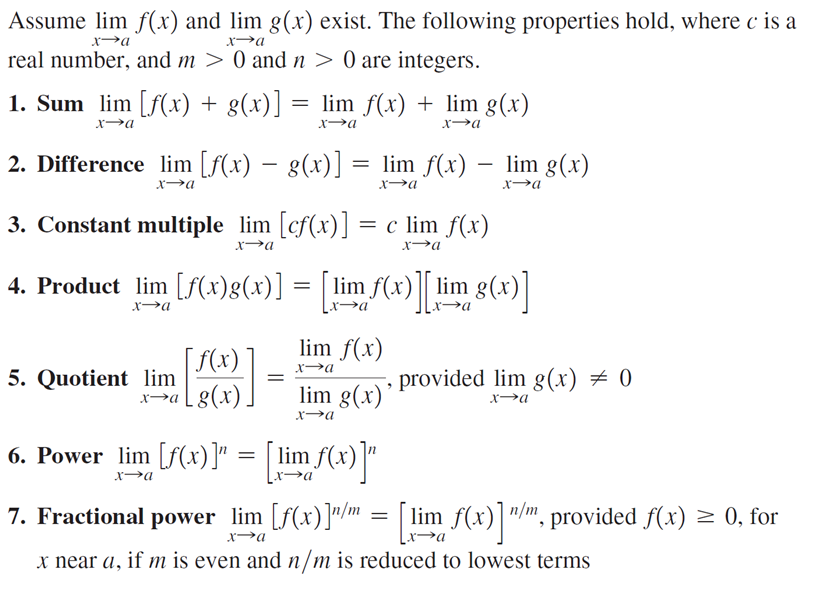


Solution: As x approaches 2 from the left, approaches 4. Therefore,

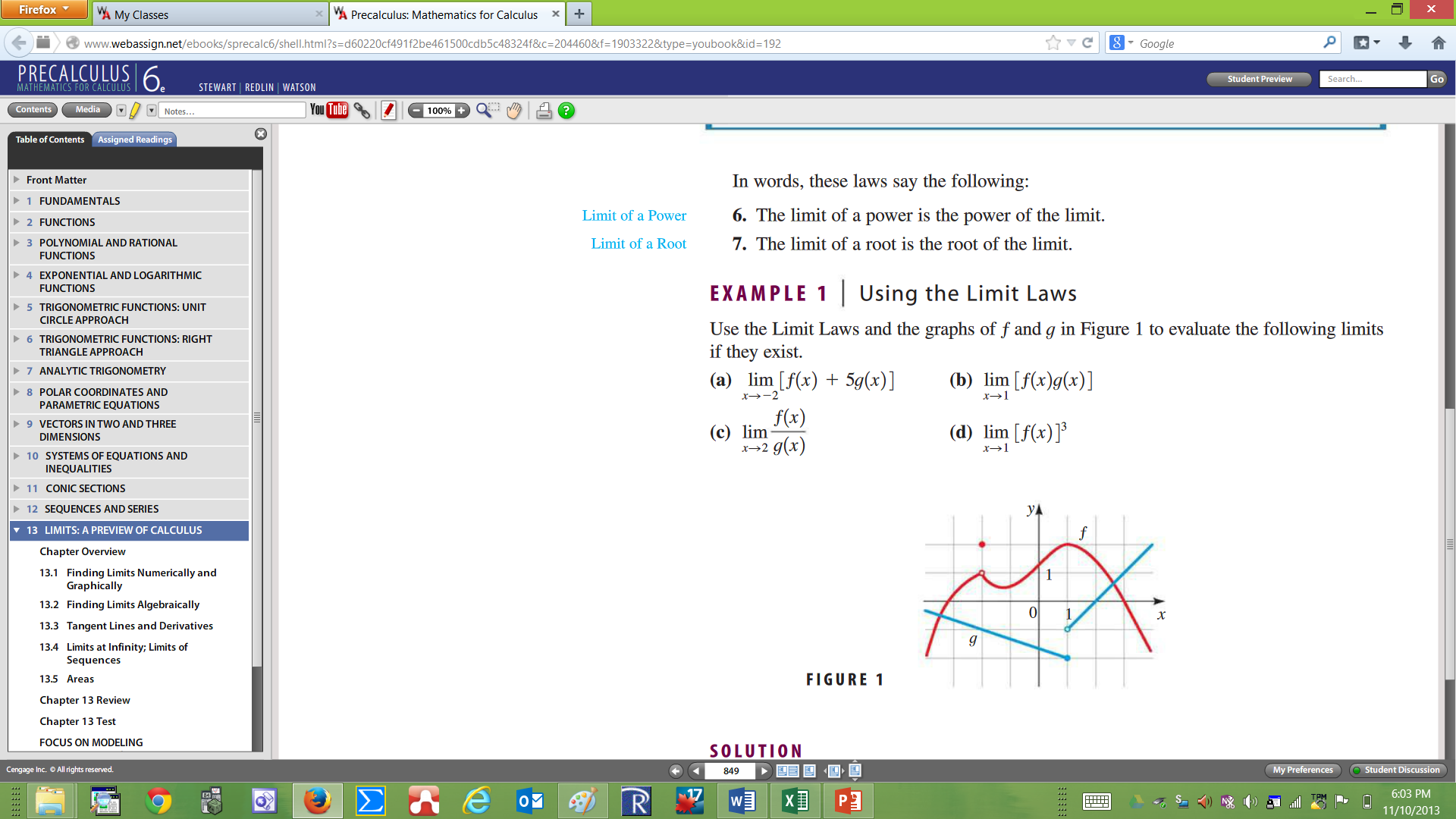
As x approaches 2 from the right, approaches 1. Therefore,

does not exist (DNE) because . (The right limit differs than the left limit)

Finding Limits Algebraically (Limits Laws)



**Example 5**: **Using the Limit Laws**



Solution: (a) From the graph, we see that and.

Therefore Using Law 1 (Limit of a Sum)

Using law 3 (Limit of a Constant Multiple)

(b)From the graph, we see that, but does not exist because the limit from the right does not equal the limit from the left; so does not exist. We cannot use Law 4 (Limit of a Product)

(c)From the graph, we see that, but. We cannot use Law 5 (Limit of

a Quotient). The limit does not exist because the denominator approaches 0.

(d)From the graph, we see that, so using Law 6 (Limit of a Power), we get

**Example 6**: **Finding Limit by Canceling Common Factors.** Find **.**

**Solution:**

**Example 7**: **Finding Limit by Simplifying.** Find **.**

**Example 8**: **Finding Limit by Rationalizing.** Find

**Solution:**  where we have multiplied numerator and denominator by the conjugate of the numerator. This makes the numerator a difference of squares, so .

**Example 9**: **Comparing Right and Left Limits.** Find **.**

**Solution:** Since for and for, for and for. Since the right-hand and left-hand limits are different, it follows that does not exist.

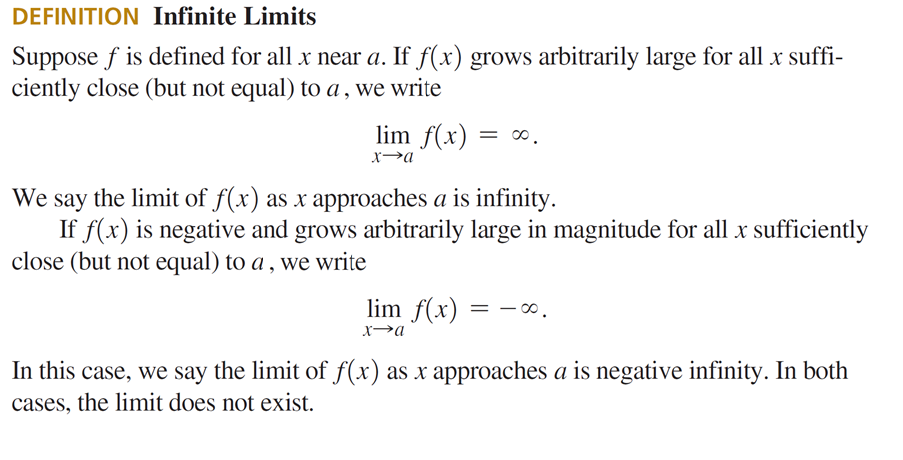
**Example 10**: **The Limit of a Piecewise Function.** Findif it exist, where

**Solution:** Since,.

Since,.

Since the right and left-hand limits are equal, the limit exist, and **.**

**Infinite Limits**

****

**Example 4**: **An Infinite Limit** Find and

Solution: The graph of in figure below shows that as x approaches 1 from either side, the values of grow arbitrarily large, so the limit does not exist and we write

As x approaches −1 from either side, the values of are negative grow arbitrarily large in magnitude, so the limit does not exist and we write



