Line Integrals Worksheet

1) Evaluate the line integrals (a) by direct integration along any path, (b) by finding the potential.

a) $\int_{(-1,2)}^{(1,3)} y^2 dx + 2xy dy$ **Ans:** 13

b) $\int_{(0,1)}^{(\pi,-1)} y\sin x dx - \cos x dy$ **Ans:** 0

2) Integrate by any method.

a) $\oint_c y\sin x dx - \cos x dy$ **(*c* any path)** **Ans:** 0

b) $\int_{(0,0)}^{(3,2)} 2xe^y dx + (x^2 e^y + 2y) dy$ **Ans:** $9e^2 + 4$

c) $\int_{(2,-2)}^{(-1,0)} 2xy^3 dx + (3y^2 x^2 + 1) dy$ **Ans:** 34

3) Find the work done by the force $\vec{F} = \left\langle \frac{y}{x^2+y^2}, -\frac{x}{x^2+y^2} \right\rangle$ ***N*** **on a particle that moves from the point** $P = (0,2)$ **to the point** $Q = (3,3)$ **with distance measured in meters.** **Ans:** $\pi/4$ *J*

4) Find the circulation of the velocity field $\vec{v} = \left\langle e^y + ye^x, xe^y + e^x \right\rangle$ **of a fluid from the point (1, 0) to the point (-1, 0), without finding the potential function,**

a) along the *x*-axis. **Ans:** -2

b) along the semicircle $\sqrt{1-x^2}$ **.** **Ans:** -2

5) Find the flux of the velocity field $\vec{v} = \langle y, x \rangle$ **of a fluid with density** $\delta = 1$ **from the point (1, 0) to the point (-1, 0), without finding the potential function,**

a) along the *x*-axis. **Ans:** 0

b) along the semicircle $\sqrt{1-x^2}$ **.** **Ans:** 0

6) What is the flux of the velocity field $v = xy\mathbf{i} + x^2y^2\mathbf{j}$ across the quarter ellipse C : $x = 4\cos t$, $y = \sin t$, $0 \xrightarrow[t]{} \frac{1}{2}\pi$? **Ans:** $\frac{148}{15}$

7) Find the flux of the velocity field $v = 2y\mathbf{i} - y^3\mathbf{j}$ across C: $y = e^x$, $0 \xrightarrow[x]{} 2$

Ans: $e^4 + \frac{1}{3}e^6 - \frac{4}{3}$