Volumes of Revolution

1 Disks Method

a) $V = \pi \int_a^b f(x)^2 dx = \pi \int_a^b y^2 dx \quad for\ y = f(x)$

ex. Find the volume of the solid of revolution when the area, in the first quadrant, bounded by $y^2 = 2x$ and the lines $x = 2$ and $y = 0$ is rotated about the x-axis.

$V = \pi \int_a^b y^2 dx = \pi \int_0^2 2x\, dx = \pi x^2 |_0^2 = 4\pi$ **Cu. units**

b) $V = \pi \int_c^d g(y)^2 dy = \pi \int_c^d x^2 dy \quad for\ x = g(y)$

ex. Find the volume of the solid of revolution when the area bounded by $y = 2x^2$ and the lines $x = 0$ and $y = 2$ is rotated about the y-axis.

$V = \pi \int_c^d x^2 dy = \pi \int_0^2 \frac{y}{2} dy = \frac{\pi}{4} y^2 |_0^2 = \pi$ **Cu. units**

2 Washers Method

a) $V = \pi \int_a^b f(x)^2 - g(x)^2 dx$

ex. Find the volume of the solid of revolution when the area bounded by $g(x) = x^2$ and $y = x$ is rotated about the x-axis.

$V = \pi \int_0^1 (x)^2 - (x^2)^2 dx = \pi \left(\frac{1}{3} - \frac{1}{5}\right) = \frac{2\pi}{15}$ **Cu. units**

b) $V = \pi \int_c^d f(y)^2 - g(y)^2 dy$

ex. Find the volume of the solid of revolution when the area bounded by $y = x^2$ and $y = x$ is rotated about the y-axis.

$V = \pi \int_0^1 (\sqrt{y})^2 - (y)^2 dy = \frac{\pi}{6}$ **Cu. units**

3 Cylindrical Shells

a) $V = 2\pi \int_a^b x f(x)\, dx = 2\pi \int_a^b xy\, dx \quad for\ y = f(x)$

ex. Find the volume of the solid of revolution when the area bounded by $y = x^2$ and $y = x$ is rotated about the y-axis.

$V = 2\pi \int_0^1 x(x - x^2) dx = \frac{\pi}{6}$ **Cu. Units**

b) $V = 2\pi \int_c^d y g(y)\, dy = 2\pi \int_c^d yx\, dy \quad for\ x = g(y)$

ex. Find the volume of the solid of revolution when the area bounded by $g(x) = x^2$ and $y = x$ is rotated about the x-axis.

$V = 2\pi \int_0^1 y(\sqrt{y} - y) dy = \frac{2\pi}{15}$ **Cu. units**

Homework:

1) Find the volume of the solid generated when the area bounded by $y = x^2$ and the lines $y=0$ and $x=2$ is rotated about the x-axis by (a) Using disks, (b) Using shells

2) Find the volume of the solid generated when the area bounded by $y = \sqrt{x}$ and the line $y = x$ is rotated about the y-axis by (a) Using disks, (b) Using shells

3) What is the volume of the solid generated when the region between y = x and $y = 3x - x^2$ is rotated about the y-axis?

4) What is the volume of the solid generated when the smallest region between y = x and $y = \sqrt{4-x^2}$ is rotated about the x-axis?

5) Find the volume of the solid obtained by rotating about the line $x=4$ the area between the curves $f(x) = x^2$ and $x = 0$ from y = 1 to y = 2.

Answers:

1) D: $\pi\int_0^2 x^4 dx = 32\frac{\pi}{5}$ **S:** $2\pi\int_0^4 y\left(2-\sqrt{y}\right)dy = 32\frac{\pi}{5}$

2) D: $\pi\int_0^1 y^2 - y^4 dx = 2\frac{\pi}{15}$ **S:** $2\pi\int_0^1 x\left(\sqrt{x}-x\right)dx = 2\frac{\pi}{15}$

3) S: $2\pi\int_0^2 x\left(\left(3x-x^2\right)-x\right)dx = 8\frac{\pi}{3}$

4) D: $\pi\left[\int_0^{\sqrt{2}} x^2 dx + \int_{\sqrt{2}}^2 4-x^2 dx\right] = \frac{8\pi}{3}\left(2-\sqrt{2}\right)$ **S:** $2\pi\int_0^{\sqrt{2}} y\left(\sqrt{4-y^2}-y\right)dy = \frac{8\pi}{3}\left(2-\sqrt{2}\right)$

5) D: $\pi\int_1^2 4^2 - \left(4-\sqrt{y}\right)^2 dy = \frac{\pi}{6}\left(64\sqrt{2}-41\right)$