Parametric Equations Worksheet

1) Find parametric equations for the equation that traces a circle of radius 2 with $0 \le t < 2\pi$ and the following conditions:
a) starting at the point $(0, 2)$ with counterclockwise orientation.
b) starting at the point $(-2, 0)$ with clockwise orientation.
c) starting at the point $(0, -2)$ with counterclockwise orientation.

2) Find the values of t that will trace a circle (once) with parametric equations
a) $x = cos(3t); \ y = sin(3t)$
b) $x = cos(t/3); \ y = sin(t/3)$

3) Eliminate the parameter to find the Cartesian equation. Indicate the direction of increasing t. Graph the equation by hand. Check your answer with your grapher.
a) $x = 2t + 4, \ y = t - 1 \quad -3 \le t \le 3$
b) $x = 1 - 2t, \ y = t^2 + 4 \quad\quad 0 \le t \le 3$
c) $x = 2\,cos(t), \ y = \frac{1}{2}sin(t)$
d) $x = e^t, \ y = e^{-t}$
e) $x = 2 + cost, \ y = 3 + sint \ \ 0 \le t \le 2\pi$
f) $x = 1/t, \ y = ln(t) \quad 1 \le t \le e$

4) Suppose a red ant and a black ant are on the x-y plane. If the red ant position at time t, in seconds, is given by $x = 10t, \ y = t^2$, and the black ant position at time t is given by $x = 2t, \ y = t - 6$. Find algebraically the intersection and collision points of the path, if any.

5) At time t (hours) one plane has position $x = 30t, y = 20t^2$ with distances measured in miles, and another plane at the same altitude has position $x = 40t - 40, \ y = 120t - 160$. Find algebraically the intersection and collision points of the path, if any.

6) Find the equation of the tangent line to the curve $x = e^{\sqrt{t}}, \ y = t - ln(t)$ at $t = 4$.

7) Find $\frac{d^2y}{dx^2}$ if $x = 4sin\,(t), \ y = cos(2t)$ at $t = \frac{\pi}{3}$ without eliminating the parameter.

8) Find the values of t at which the curve $x = \frac{t^3}{3} - t, \ y = t^2 + t$ has
a) horizontal tangent lines, b) vertical tangent lines.

9) Find analytically the arc length of:
a) $x = 2t, y = \frac{2}{3}t^{3/2}$ in $1 \le t \le 2$.
b) $x = e^t cos(t), y = e^t sin(t)$ in $0 \le t \le 1$.

10) A gun is fired from a bunker of $2m$. deep with an initial velocity of $500m/s$ at an angle of $\frac{\pi}{6}$.
a) Find the parametric equations of the position of the projectile
b) Find the range of the projectile to the nearest meter.
c) Find the maximum height of the projectile to the nearest meter.

11) Sally hits a softball $3ft$. above the ground at a 30° angle with respect to the ground and a velocity of 80 ft/s. Will the ball clear a $6ft$ tall wall that is 168 ft away?
Check your answers graphically. Be careful about rounding errors.

12) Sammy hits a ball hits a baseball when it is $2.5ft$ above the ground. The ball leaves his bat at an angle of 29° from the horizontal with an initial velocity of 136 ft/s.
a) Find the maximum height of the ball and the velocity of the ball at that point.
b) The horizontal distance the ball travels before it is caught $8ft$ into the air. Round to two the nearest feet. Check your answers graphically.