Convergence Tests

1) Divergence Test

If $\lim_{k\to\infty} u_k \neq 0$, then $\sum u_k$ diverges.

NOTE: If $\lim_{k\to\infty} u_k = 0$, then $\sum u_k$ may or may not converge.

2) Integral Test

Let $\sum_{k=1}^{\infty} u_k$ be a series with non negative terms, and let $f(x)$ be the function that results when k is replace by x in the formula for u_k . If $f(x)$ is non increasing and continuous for $x \geq 1$, then $\sum_{k=1}^{\infty} u_k$ and $\int_1^{\infty} f(x)$ both converge or diverge.

NOTE: Use when $f(x)$ is easy to integrate.

3) Comparison Test

Let $\sum a_k$ and $\sum b_k$ be series with non negative terms such that $a_k \leq b_k$.

If $\sum b_k$ converges, $\sum a_k$, and if $\sum a_k$ diverges, $\sum b_k$ diverges.

NOTE: Use this test as a last resort. Some other tests are easier to use.

4) Limit Comparison Test

Let $\sum a_k$ and $\sum b_k$ be series with non negative terms such that $\rho = \lim_{k\to\infty} \frac{a_k}{b_k}$

If $0 < \rho < +\infty$, then both series converge or diverge. If $\rho = 0$ or $\rho = \infty$, the test fails.

NOTE: Require some skill in choosing the series $\sum b_k$ for comparison.

5) Alternating Series Test

If $a_k > 0$ for all k, the series $\sum_k (-1)^k a_k$ or $\sum_k (-1)^{k+1} a_k$ converges if

a) a_k is non increasing b) $\lim_{k\to\infty} a_k = 0$

Definition of Absolute Convergence

Let $\sum a_k$ have some positive and some negative terms. If $\sum |a_k|$ converges then $\sum a_k$ converges absolutely (in absolute value). If $\sum |a_k|$ diverges but $\sum a_k$ converges then $\sum a_k$ converges conditionally.

6) Ratio Test

Let $\sum u_k$ be any series such that $\rho = \lim_{k\to\infty} \frac{|u_{k+1}|}{|u_k|}$

a) the series converges absolutely if $\rho < 1$.
b) the series diverges if $\rho > 1$ or $\rho = \infty$.
c) No conclusion if $\rho = 1$.
Note: Try this when u_k involve factorials and k_{th} powers

7) Root Test

Let $\sum u_k$ be any series such that $\rho = \lim_{k\to\infty} \sqrt[k]{|u_k|}$

a) the series converges absolutely if $\rho < 1$.
b) the series diverges if $\rho > 1$ or $\rho = \infty$.
c) No conclusion if $\rho = 1$.
Note: Try this when u_k involve k_{th} roots.