MAC 2311 Hybrid Calculus I

Sections **4.3,4.5**

**Increasing/decreasing functions**

A continuous function is increasing (inc) on the interval I if on I.

A continuous function is decreasing (dec) on the interval I if on I.

Eg8. Find the intervals where is increasing/ decreasing

Ans: from sign chart, inc (-∞,1) U(2,∞); dec (1,2)

**First Derivative Test (FDT) for Relative Extrema**

Suppose is differentiable at every point on an Interval I that contains the critical value c (except perhaps at c itself).

If changes sign from positive to negative as x increases through the value c, then is a local maximum at c.

If changes sign from negative to positive as x increases through the value c, then is a local minimum at c.

If does not change sign through c, then has no local extrema at c.

Eg9. Use the FDT to find the relative extrema of Ans: (0,1) neither max or min; (1,67/60) max; (2,11/15) min

**Second Derivative Test (SDT) for Relative Extrema**

Suppose is continuous on the open interval I containing c with (at CV)

If then has a local minimum at c.

If then has a local maximum at c.

If then the test fails. If the test fails, the first derivative test must be used to find local extrema.

Eg10. Use the SDT to find the relative extrema of

Ans: CV at *x*=0,1,2, so, test fails; , CD or rel max; , CU or rel min. Since SDT fails at x=0, the point (0,1) is neither relative max nor min by the FDT( see Eg9)

Notes: If (c, *f*(c)) is a relative extremum of, then.

The converse is not true. If, you may or may not may or may not have a relative extremum.

**Concavity**

From the intuitive stand point, an arc of a curve is said to be concave up if it has the shape of a cup and concave down if it has the shape of a cap.

If on an open interval I, then the graph of is concave up (CU) on I.

If on an open interval I, then the graph of is concave down (CD) on I.

**Test for Concavity (Inflection Points (IP))**

Suppose exists on the interval I.

If on I, then is concave up on I.

If on I, then is concave down on I.

If (c, *f*(c)) is a point on I at which, and changes at c (changes concavity), then (c, *f*(c)) is an inflection point of

Notes: If (c, *f*(c)) is an inflection point of, then.

The converse is not true. If, you may or may not may or may not have an inflection point.

Eg11. Find the inflection points of Ans:, but there is no IP

Eg12. Find the inflection points of Ans: (0,0), (2,-4)

**Curve Sketching**

Given; (stands for the function in the numerator and  the denominator)

1. Domain of the function: Possible values of *x.*
2. Intercepts of the function: *xint* solve *N(x)* = 0; *yint= f(0*) .
3. Symmetry: If the function is even (symmetric about the *y-axis*); if the function is odd (symmetric about the *origin*);
4. Asymptotes of the function:

a) Vertical (VA): solve*.* If *x=a* is a vertical asymptote, evaluate  and to find the direction of the curve at the asymptote.

b) Horizontal (HA): Evaluate. This also tells you the behavior of the graph far to the right and left.

1. Interval of Increase/ Decrease Use the first derivative test to find the intervals where *f* *'(x)* >0 (increasing) and where *f* *'(x)* <0 (decreasing). Test *f '(x)* in regions given by zeros, CP and VA in the domain of the function.
2. Local Maximum and Minimum Values: Find the critical points (CP) by finding points where *f* *'(x)* = 0 or *f '(x)* is undefined. Find the *y* coordinate by evaluating *f(x)* at those points. Remember that not all critical points are relative extrema.
3. Concavity and Points of Inflection: Solve *f* *"(x)* = 0 for *x*, to find the possible inflection points. Inflection points are points where concavity changes. Not all points where *f* *"(x)* = 0 are inflection points.

Find where *f (x)* is concave up (CU) or concave down (CD) by testing *f ''(x)* in regions given by zeros, CP and VA in the domain of the function. If you evaluate at the CP, you obtain  (CU/relative min); (CD/ relative max).

1. Sketch the curve without the use of a graphics calculator. Check with a grapher.

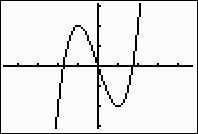
Note: Start your preliminary sketch as you find the information about the graph in parts A-G.

Examples:

1. Sketch the graph *of f(x)* = *x3 -*3*x*

1. Domain: all *x.*
2. *xint*  *x(x2-*3*)* = 0  *x* = 0, *x=*; *yint* = 0.
3. the function is odd (symmetric about the *origin*)
4. No asymptotes. = =
5. Increasing when *y'* = *3x2* -3 > 0 on **, and decreasing when

*y'* = *3x2* -3 < 0 on (-1,1).

1. Critical numbers at *y'* = *3x2* -3 = 0 *(x* = *-*1); *(x* = l*,*). ** is changes from positive to negative at *x* = *-*1 ( I(-1)=2 is a local maximum) and changes from negative to positive at *x* = 1 ( f(1)=-2 is a local minimum)
2. = 0 at (0,0). > 0 for *x>*0 (CU); < 0 for *x*<0 (CD). Since concavity changes, (0,0) is an IP. |1> 0 (CU/relative min.); |-1< 0 (CD/relative max.)
3. 

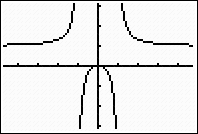
2. Sketch the graph of 

1. Domain 
2. *xint*  *x* = 0; *yint* = 0.
3.  the function is even (symmetric about the *y-axis*)
4. VA, , ==; ==HA, =1 =1
5. Increasing when  >0 on, and decreasing when

 <0 on,

1.  = 0 *(x* = 0*,y* = 0); undefined (not in the domain of *f(x)*), local maximum at *f*(0)=0, no local minimum.
2. **; no IP. *y''* = > 0 for *x>*0 for *x* <-1 or x >1 (concave up)

*y"* = <0 for -1<x<1, (concave down).

1. 

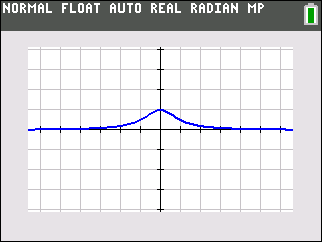
3. Sketch the graph of

1. Domain: all *x*
2. *x*int none, *y*int =1
3. Symmetry: even
4. VA none; HA
5. Interval of increase/decrease , CP *x* =0

for *x* < 0; for *x* > 0, inc(-∞,0), dec (0,∞)

1. Concavity and Inflection points:

CU (-∞,-1/√3) U (1/√3, ∞); CD (-1/√3,1/√3), IP ( ±1/√3, ¾)

1. Graph (label all points)
2. 

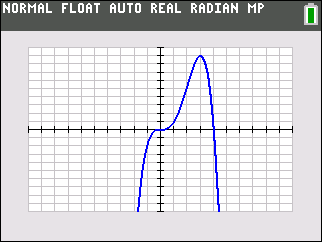
4. Sketch the graph of

1. Domain: all *x*
2. *x*int =0,4, *y*int =0
3. Symmetry: none
4. VA none; HA none
5. Interval of increase/decrease , CP (0,0), (3,27)

in (-∞,3); in (3,∞), inc(-∞,3), dec (3,∞)

1. Concavity and Inflection points:

CU (0,2); CD (-∞,0) U (2, ∞); IP ( 0,0), (2, 16)

1. Graph (label all points) Scale: *x*=1, y=3
2. 

**Exercises:**

1. Find the critical points
2. Test for symmetry
3. Find the intervals on which *f* is increasing or decreasing
4. Find the intervals where *f* is concave up or concave down.
5. Find the inflection points if any.
6. Use the information in parts a-e to sketch the graph. Check with a grapher.

1) 

2) 

3) 

4) 

5) 

6) 

7) 

8) 

9) 

10) 

**Answers:**

1. Inc. ; Dec. ; local max ; local min 

CU (; CD ; IP 

1. Inc. ; Dec. ; local max ;

local min ; CU ; CD ; IP 

1. Inc. ; Dec. ; local max ;

local min ; CU ; CD ;

IP , 

1. Inc. ; Dec. ; local max ; local min 

CU (; CD ; IP 

1. Inc. ; Dec. ; local max ;

local min ; CU ; CD ;

IP 

1. *y*-int :0; *x*-int :0; VA: *x=*1; HA: *y* =1; Dec: ; CU ; CD ; IP: none
2. *y*-int :-1/9; VA: ; HA: ; Inc. ; Dec: ; local max ; CU ; CD ; IP: none
3. *y*-int :1; *x*-int :1; VA: **; HA:; Dec: ; CU ; CD ; IP: none
4. *y*-int :0; *x*-int :0; VA: None; HA: None ; Inc: ; Dec: ; local max ; local min ; CU ; CD ; IP: ;;
5. *y*-int :0; *x*-int :0; Inc: ; Dec: ; local max ; local min ; CU ; CD ; IP: ;;