MAC 2311 Hybrid Calculus I

Sections **4.1**

**Derivatives and Graphs**

**Critical Points (values)**

If the point (c, *f*(c)) is in the domain of the function$ f$, (c, *f*(c)) is called a critical point (CP) of $f $if $f^{'}\left(c\right)=0$ or $f'(c)$ does not exist (DNE). The number c is called a critical value (CV).

Eg1. Find the CPs of $f\left(x\right)=x^{3}/3-3x^{2}/2+2x+1$ Ans: (1,11/6), (2,5/3)

Eg2. Find the CPs $g\left(x\right)=(x-1)^{2/3}$ Ans: (1,0)

Eg3. Find the CPs $h\left(x\right)=1/(x+1)$ Ans: None

Eg4. Find the CPs $I\left(x\right)=xe^{-x}$ Ans: (1,1/e)n

Eg5. Find the CPs $j\left(x\right)=\frac{x-1}{x^{2}-x+1}$ Ans: (0,-1), (2,1/3)

**Relative (local) Extrema**

If $f$ has a relative or local extrema (relative max/min) at the point (c, *f*(c)), and $f'(c)$ is defined, then$ f^{'}\left(c\right)=0$. The converse is not true. If the derivative is zero, you may or may not may or may not have a relative extrema.

Eg6. $f\left(x\right)=x^{2}+1$ Ans: $f'\left(0\right)=0$ a local min.

Eg7. $g\left(x\right)=x^{3}$ Ans: $g'\left(0\right)=0$ neither max nor min.

**Absolute Extrema**

If $f$ is a continuous function on the interval [a, b], the absolute extrema (abs max/min) will occur either at the critical points or at the end points.

Eg8. Find the Absolute Extrema of $f\left(x\right)=x^{2}-1$ in [-1, 2] Ans: abs max (2,3), abs min (0,-1)

If $f$ is a continuous function on the interval (a, b), the absolute extreme values (if they exist) will occur at interior points of the interval.

Eg9. Find the Absolute Extrema of $f\left(x\right)=x^{2}-1$in(-1, 2) Ans: abs max none, abs min (0,-1)

Eg10. Find the Absolute Extrema of $g\left(x\right)=2\cos(\left(x\right))+sin⁡(2x)$ in [0, π/ 2] Ans: abs max (π/6,$\sqrt{3}+\frac{\sqrt{3}}{2})$ abs min (π/2,0)

Eg11. Find the Absolute Extrema of $h\left(x\right)=ln⁡(x^{2}+x+1$ in [-1, 1] Ans: abs max (1,ln3), abs min (1/2,ln(3/4))