**Exponential Functions Handout**

**The following pages contain the Exponential Function, and derivatives of the Exponential function. These topics are not in your interactive e-book, but are required for this course. Please read carefully. This material will be part of Test #3**

**Exponential Functions**

In this section we will study an important class of functions that occurs in a wide variety of applications. Exponential functions are those which have the independent (or input) variable in the exponent. For example  or  . In linear functions when the input variable increases by fixed steps the function values change by adding a constant. But in exponential functions the function values change by multiplying by a constant. The following examples will demonstrate this.

**Example 1**.

In the 1990s the population of the state of Florida increased about 3% per year. In 1990 the population was about 14 million. Let P(t) be the population t years after 1990. Find a formula for P(t).

**Solution**: The fact that the population increases by 3% each year can be described by the relationship

Population next year = population this year + 0.03\* population this year which can be written as

Population next year = 1.03\*population this year.

This allows us to make the table

|  |  |  |
| --- | --- | --- |
| year | t | P(t) |
| 1990 | 0 | 14 |
| 1991 | 1 | 14\*1.03 |
| 1992 | 2 | (14\*1.03)\*1.03=14\*1.032 |
| 1993 | 3 | (14\*1.032)\*1.03=14\*1.033 |

From the table we can see that.

**Example 2**.

Radioactive elements decay into more stable elements as time goes by. The rate of the decay is commonly given in terms of the half-life of the element. The half-life is the time required for one half of the material to decay. Plutonium-238 is a radioactive element that has a half-life of 88 years. Suppose that a power plant has 50 pounds of plutonium-238 to dispose of. Write a formula for a function that gives the amount of the plutonium-238 remaining after t years.

**Solution**:

The key is that at the end of 88 years the amount of Plutonium-238 that remains will be one half of what it was at the beginning of the 88 year period. To see the pattern it is helpful to look at the table in 88 year steps. Let P(t) be the amount that remains after t years.

|  |  |
| --- | --- |
| t | P(t) |
| 0 | 50 |
| 88 | 50\* |
| 176 | (50\*)\* = 50 |
| 264 | (50)\* = |

Notice that the exponent on  in each row is just the multiple of 88 used for t. So we can write the exponent as . Thus the function is  .

Now that we have reviewed exponential functions briefly we will turn our attention to the derivative of these functions.

**Derivative of ex**

To find the derivative of *f*(*x*) = *e*x we will use the definition of the derivative.





Using a law of exponents  and so the above limit becomes



The numerator can be factored and we get



As an example we will look at the case where b is 2. In this case



To get an estimate of the limit above we will look at a table of values.

|  |  |
| --- | --- |
| h |  |
| 0.1 | 1.052 |
| 0.01 | 1.005 |
| 0.001 | 1.0005 |
| 0.0001 | 1.0001 |
| 0.00001 | 1.000005 |

This gives the formula



In algebra you learned that e is called the natural base. Because of the simplicity of the derivative formula when the base is e you can finally see why it is such an important and special base.

**Example 3**

Find the equation of the tangent line to y = ex at x = 0.

**Solution**: The point on the curve y = ex at x = 0 is (0, e0) = (0,1). The slope of the tangent line is  . At x = 0 the slope = e0 = 1. Thus the equation of the tangent line is y – 1 = 1(x – 0) or simplified we get y = x + 1.

|  |  |
| --- | --- |
| The graph to the right shows the graphs of y = ex and the tangent line.  The tangent line gives a useful approximation  ex ≈ x + 1 for x close to 0. |  |

**Example 4** Find the derivative of f(x) = x2ex.

**Solution**: Using the product rule gives



.

In most applications the function involved is not just ex but rather instead functions which have the form . As we will see this type of function can be differentiated using the Chain Rule. Here are the details.

Suppose that y =  . Let u = f(x). Then y =  . By the Chain Rule





This result is worth emphasizing.



**Example 5** Find the derivative of .

**Solution**: Using the formula in the box above we get





**Example 6**.

The temperature of a pie taken from an oven at a temperature of 350˚ F can be modeled by the function  where t is the time (in minutes) since the pie was taken from the oven. What is F(60) and F′(60) ? Write a sentence interpreting the meaning of each in context.

**Solution**: F(60) = .







After 60 minutes the temperature of the pie is 158˚ and decreasing at the rate 1.66 degrees per minute.

**Derivative of bx**

To finish this discussion of derivatives for exponential functions we will find a formula the derivative of *f*(*x*) = *bx*. To do this we need a quick review of the natural logarithm, ln(*x*).

**Definition**: For x > 0 ln(*x*) is the exponent that must be put on *e* to get *x*. Or put into symbols we have.

One special value we get from this definition is ln(1) = 0 since 0 is the exponent we put on ‘*e’* to get 1. In symbols  so we have ln(1) = 0.

Another consequence of the definition is ln(*ex*) = *x*. The exponent you put on ‘*e’* to get ex is *x*.

There is one more property that follows from the definition that we will use. We need to know another way to write ln (*bc*). To do this first note that ln (*bc*) is the exponent on ‘*e’* that gives *bc*. We can write  from the definition of the natural logarithm. So we have. Using a law of exponents we get. We now see the exponent you put on ‘*e’* to get *bc*. Therefore ln(*bc*) = *c*ln(*b*).

With this property we are ready to find the derivative of bx. First write *b*x as

. Then apply our rule for differentiating *ef(x)* to get

 since is constant

But recalling that = we have the formula



**Example 7** Find .

**Solution**: Using the formula above, 

Suppose that, The derivative will be given by



**Example 8** Find the derivative of .

**Solution**: Using the formula above, 