Related Rates:

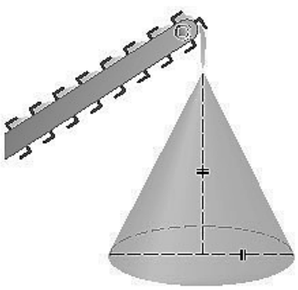
1) A balloon is inflated and its volume increases at a rate of 2 cm^3/min. At what rate is the radius of the balloon changing when the radius is 5 cm?

Solution:

Given: $\frac{dV}{dt} = 2\ \text{cm}^3/\text{min}.$ Asked: $\frac{dr}{dt}|_{r=5}$

Since $V = \frac{4}{3}\pi r^3$, $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$, so $2 = 4\pi 5^2 \frac{dr}{dt}$, or $\frac{dr}{dt} = \frac{1}{50}$cm/min.

2) Gravel is being dumped from a conveyor belt and accumulates in a conical pile with radius that is always 3 times its height. If gravel falls from the belt at a rate of 100ft^3/min, how fast is the height of the gravel pile changing when the pile is 10 ft. high? The volume of a cone is $V = \frac{1}{3}\pi r^2 h$. Your answer should be exact (No decimals). Show units.



Solution:

Given: $r = 3h$, $\frac{dV}{dt} = 100\text{ft}^3/\text{min}.$ Asked: $\frac{dh}{dt}|_{h=10}$

Since V=1/3 $\pi r^2 h$, we get V=3 πh^3.

So $\frac{dV}{dt} = 9\pi h^2 \frac{dh}{dt}$, or $100 = 9\pi 10^2 \frac{dh}{dt}$, so $\frac{dh}{dt}|_{h=10} = \frac{1}{9\pi} ft/min$

3) A 13-ft ladder is leaning against a vertical wall. If the foot of the ladder is pulled away at a rate of ½ ft/s, how fast is the top of the ladder sliding when the lower end of the ladder is (a) 4 ft. from the wall? (b) 5 ft. from the wall? (c) 6 ft. from the wall? Find the exact answer, and the answer rounded to 4 decimal places.

Solution:

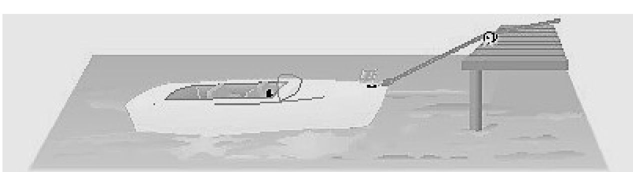
Given: length=13ft., $\frac{dx}{dt} = \frac{1}{2} ft/s.$ Asked: $\frac{dy}{dt}|_{x=4,5,6}$

$x^2 + y^2 = 13^2$, so $2x\frac{dx}{dt}+2y\frac{dy}{dt} = 0$ so $\frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt}$

$\frac{dy}{dt}|_{x=4} = -\frac{4}{\sqrt{13^2-4^2}}\frac{1}{2} = -\frac{2}{\sqrt{153}} \approx -.1617 ft/s$; $\frac{dy}{dt}|_{x=5} = -\frac{5}{\sqrt{13^2-5^2}}\frac{1}{2} = -\frac{5}{2\sqrt{144}} \approx -.2083 ft/s$

$$\frac{dy}{dt}|_{x=6} = -\frac{6}{\sqrt{13^2-6^2}}\frac{1}{2} = -\frac{3}{\sqrt{133}} \approx -.2601 ft/s$$

4) A boat is pulled into a dock by a rope attached to the bow of the boat and passing through a pulley on the dock that is 3 m higher than the bow of the boat. If the rope is pulled in at a rate of 2 m/s how fast is the boat approaching the dock when 5 m of rope are left to pull in? Your answer should be exact (No decimals).



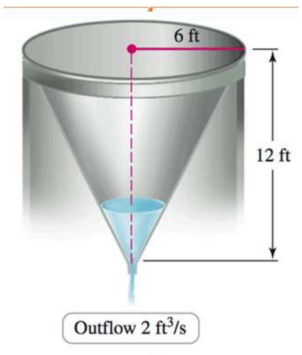
Solution:

Given: $y = 3$; $\frac{dz}{dt} = 2m/s$ Asked: $\frac{dx}{dt}|_{z=5}$

Since $x^2 + 3^2 = z^2$, $2x\frac{dx}{dt} + 0 = 2z\frac{dz}{dt}$, so $\frac{dx}{dt} = \frac{z}{x}\frac{dz}{dt}$

$$\frac{dx}{dt}|_{z=5} = \frac{5}{\sqrt{5^2-3^2}}2 = \frac{5}{2} m/s$$

5) An inverted conical water tank with a height of 12 ft. and a radius of 6 ft. is drained through a hole in the vertex at a rate of 2ft/s. What is the rate of change of the water depth when the water depth is 3 ft? The volume of a cone is $V = \frac{1}{3}\pi r^2 h$. Your answer should be exact (No decimals). Show units.



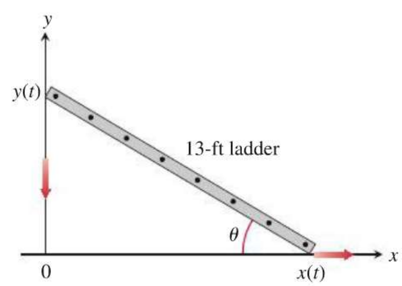
Solution:

Given: $h = 12ft,\ r = 6ft\,, \frac{dV}{dt} = -2ft/s.$ Asked: $\frac{dh}{dt}|_{h=3}$

Since V=1/3 πr²h, by similar triangles, 6/r=12/h or r=h/2. Into V, we get V=1/3 π(h/2)²h= 1/12 πh³.

So $\frac{dV}{dt} = \frac{1}{4}\pi h^2\ \frac{dh}{dt}$, or $-2 = \frac{1}{4}\pi 9\ \frac{dh}{dt}$, so $\frac{dh}{dt} = -\frac{8}{9\pi} ft/s$

6) A 13-ft ladder is leaning against a vertical wall. If the foot of the ladder is pulled away at a rate of ½ ft/s, at what rate is the angle of inclination θ of the ladder with respect to the ground changing when the lower end of the ladder is 5 ft from the wall?

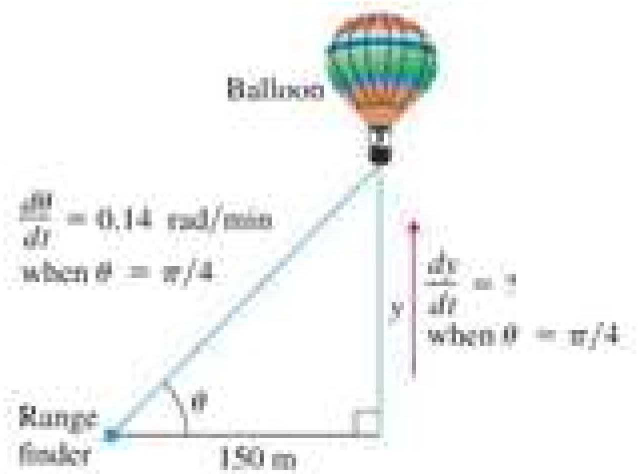


Solution:

Given: length=13ft, $\frac{dx}{dt} = \frac{1}{2} ft/s$ Asked: $\frac{d\theta}{dt}|_{x=5}$

Since $cos\,\theta\ = \frac{x}{13}, -sin\,\theta\ \frac{d\theta}{dt} = \frac{1}{13}\frac{dx}{dt}$. At $x = 5, sin\,\theta = \frac{12}{13}$, so $-\frac{12}{13}\frac{d\theta}{dt} = \frac{1}{13}\frac{1}{2}$, so $\frac{d\theta}{dt} = -\frac{1}{24} rad/s$

7) A hot air balloon rising straight up is tracked by a range finder 150ft from the lift up point. At the moment the range finder's elevation angle is π/4, the angle is increasing at a rate of 0.14 rad/min. How fast is the balloon rising at that moment?



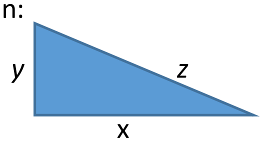
Given: $\frac{d\theta}{dt}|_{\theta=\pi/4} = 0.14 rad/min$ Asked: $\frac{dy}{dt}|_{\theta=\pi/4}$

Since $\tan(\theta) = \frac{y}{150}$, $\sec^2(\theta)\frac{d\theta}{dt} = \frac{1}{150}\frac{dy}{dt}$, so at θ=π/4, $\sqrt{2}^2(0.14) = \frac{1}{150}\frac{dy}{dt}$, or

$\frac{dy}{dt}|_{\theta=\pi/4} = 42ft/min$

8) Two ships, A and B, leave port at noon. Ship A travels north at 6.00 mph, and ship B travels east at 8.00 mph. How fast are they separating at 2:00 pm?

Solution:



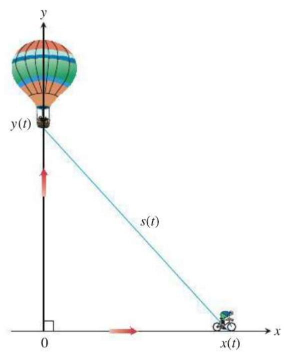
Given: $\frac{dx}{dt} = 8mph$, $\frac{dy}{dt} = 6mph$ Asked: $\frac{dz}{dt}|_{t=2}$

Since $x^2 + y^2 = z^2$, $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$, so $\frac{dz}{dt} = \frac{x}{z}\frac{dx}{dt} + \frac{y}{z}\frac{dy}{dt}$

$$\frac{dz}{dt}|_{t=3} = \frac{16}{\sqrt{16^2+12^2}} * 8 + \frac{12}{\sqrt{16^2+12^2}} * 6 = \frac{16*8+12*6}{\sqrt{16^2+12^2}} = 10\ mph$$

9) A balloon rising straight up at a constant rate of 1ft/s. Just when the balloon is 65ft above the ground, a bicycle moving at a rate of 17ft/s passes under it. How fast is the distance between the `bicycle and the balloon increasing 3s later?

Solution



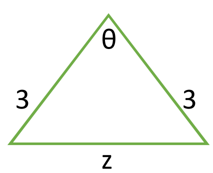
Given: $\frac{dx}{dt} = 17\frac{ft}{s}, \ \frac{dy}{dt} = 1\frac{ft}{s}$ Asked: $\frac{ds}{dt}|_{t=3}$

Since $x^2 + y^2 = s^2, \ 2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2s\frac{ds}{dt}$, so $\frac{ds}{dt} = \frac{x}{s}\frac{dx}{dt} + \frac{y}{s}\frac{dy}{dt}$

$$\frac{ds}{dt}|_{t=3} = \frac{17 * 3}{\sqrt{51^2 + 68^2}} * 17 + \frac{65 + 3}{\sqrt{51^2 + 68^2}} * 1 = 11\,ft/s$$

10) A door that is 3 ft. wide is swinging shut at a rate of 2 radians /sec. Find the rate at which the distance between the door Jamb and the end of the door's free edge is changing when the door has Pi/6 radians remaining through which to close. Your answer should be exact.

Solution:



Given: $\frac{d\theta}{dt} = -2rad/s$ Asked: $\frac{dz}{dt}|_{\theta=\pi/6}$

Using the law of cosines, $z^2 = 3^2 + 3^2 - 2 * 3 * 3\cos(\theta)$, so $2z\frac{dz}{dt} = 2 * 3 * 3sin(\theta)\frac{d\theta}{dt}$,

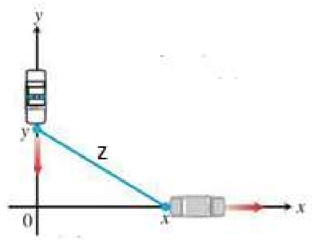
or $\frac{dz}{dt}|_{\theta=\pi/6} = -\frac{9}{z}$.

Since at $\theta = \pi/6, \ z^2 = 3^2 + 3^2 - 2 * 3 * 3\frac{\sqrt{3}}{2}$, or $z = 3\sqrt{2-\sqrt{3}}$, $\frac{dz}{dt} = -\frac{9}{z} = -\frac{3}{\sqrt{2-\sqrt{3}}}$,

$$\frac{dz}{dt} = -\frac{3}{\sqrt{2-\sqrt{3}}} \approx -5.8ft/s$$

11) A police cruiser, approaching a right-angle intersection from the north, is chasing a speeding car that has turned the coroner and is now moving straight east. When the cruiser is 0.6 mi north of the intersection and the car is 0.8 mi to the east, the police determine with radar that the distance between them and the car is increasing at 20 mph. If the cruiser is moving at 60 mph at the instant of measurement, what is the speed of the car?

Solution

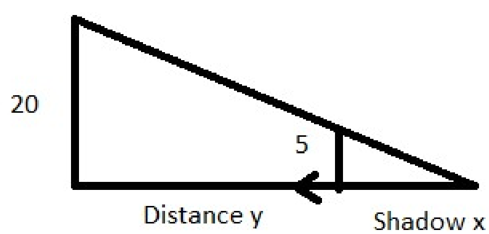


Given: $\frac{dz}{dt} = 20mph,\ \frac{dy}{dt} = -60mph, x = 0.8mi, y = 0.6mi$ Asked: $\frac{dx}{dt}|_{x=0.8}$

Since $x^2 + y^2 = z^2,\ 2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$, so $\frac{dx}{dt} = \frac{z}{x}\frac{dz}{dt} - \frac{y}{x}\frac{dy}{dt}$

$$\frac{dx}{dt}|_{x=0.6} = \frac{\sqrt{0.8^2 + 0.6^2}}{0.8} * 20 - \frac{0.6}{0.8} * (-60) = 70\ mph$$

12) A 5 ft. tall woman walks 8ft/s towards a street light that is 20ft. above the ground. What is the rate of change of the length of the shadow when she is 15 ft. from the street light? At what rate the tip of her shadow moving?



Given: $\frac{dy}{dt} = -8\frac{ft}{s}$ Asked: $\frac{dx}{dt}|_{y=15},\ \frac{dx}{dt} + \frac{dy}{dt}$

Since $\frac{5}{x} = \frac{20}{x+y}$, 5y=15x or y=3x. $\frac{dy}{dt} = 3\frac{dx}{dt}$, or $-8 = 3\frac{dx}{dt}$, so $\frac{dx}{dt} = -\frac{8}{3}\frac{ft}{s}$ for any *y*.

The tip of the shadow is moving at a rate of −(8+8/3)= −32/3 ft/s.