MAC 2311 Hybrid Calculus I

Sections 4.3-4.4 No Calculator

**Curve Sketching**

Given; (stands for the function in the numerator and  the denominator)

1. Domain of the function: Possible values of *x.*
2. Intercepts of the function: *xint* solve *N(x)* = 0; *yint= f(0*) .
3. Symmetry: If the function is even (symmetric about the *y-axis*); if the function is odd (symmetric about the *origin*);
4. Asymptotes of the function:

a) Vertical (VA): solve*.* If *x=a* is a vertical asymptote, evaluate  and to find the direction of the curve at the asymptote.

b) Horizontal (HA): Evaluate. This also tells you the behavior of the graph far to the right and left.

1. Interval of Increase/ Decrease Use the first derivative test to find the intervals where *f* *'(x)* >0 (increasing) and where *f* *'(x)* <0 (decreasing). Test *f '(x)* in regions given by zeros, CP and VA in the domain of the function.
2. Local Maximum and Minimum Values: Find the critical points (CP) by finding points where *f* *'(x)* = 0 or *f '(x)* is undefined. Find the *y* coordinate by evaluating *f(x)* at those points. Remember that not all critical points are relative extrema.
3. Concavity and Points of Inflection: Solve *f* *"(x)* = 0 for *x*, to find the possible inflection points. Inflection points are points where concavity changes. Not all points where *f* *"(x)* = 0 are inflection points.

Find where *f (x)* is concave up (CU) or concave down (CD) by testing *f ''(x)* in regions given by zeros, CP and VA in the domain of the function. If you evaluate at the CP, you obtain  (CU/relative min); (CD/ relative max).

1. Sketch the curve without the use of a graphics calculator. Check with a grapher.

Note: Start your preliminary sketch as you find the information about the graph in parts A-G.

Examples:

1. Sketch the graph *of f(x)* = *x3 -*3*x*

1. Domain: all *x.*
2. *xint*  *x(x2-*3*)* = 0  *x* = 0, *x=*; *yint* = 0.
3. the function is odd (symmetric about the *origin*)
4. No asymptotes. = =
5. Increasing when *y'* = *3x2* -3 > 0 on **, and decreasing when

*y'* = *3x2* -3 < 0 on (-1,1).

1. Critical numbers at *y'* = *3x2* -3 = 0 *(x* = *-*1); *(x* = l*,*). ** is changes from positive to negative at *x* = *-*1 ( I(-1)=2 is a local maximum) and changes from negative to positive at *x* = 1 ( f(1)=-2 is a local minimum)
2. = 0 at (0,0). > 0 for *x>*0 (CU); < 0 for *x*<0 (CD). Since concavity changes, (0,0) is an IP. |1> 0 (CU/relative min.); |-1< 0 (CD/relative max.)
3. 

2. Sketch the graph of 

1. Domain 
2. *xint*  *x* = 0; *yint* = 0.
3.  the function is even (symmetric about the *y-axis*)
4. VA, , ==; ==HA, =1 =1
5. Increasing when  >0 on, and decreasing when

 <0 on,

1.  = 0 *(x* = 0*,y* = 0); undefined (not in the domain of *f(x)*), local maximum at *f*(0)=0, no local minimum.
2. **; no IP. *y''* = > 0 for *x>*0 for *x* <-1 or x >1 (concave up)

 *y"* = <0 for -1<x<1, (concave down).

1. 

3. Sketch the graph of $f\left(x\right)=\frac{1}{x^{2}+1} $

1. Domain: all *x*
2. *x*int none, *y*int =1
3. Symmetry: even
4. VA none; HA $\lim\_{x \to \infty }\left(1/(x^{2}+1)\right)=0$
5. Interval of increase/decrease $f^{'}(x)=-2x/(x^{2}+1)^{2}$, CP *x* =0

$f^{'}\left(x\right)>0$ for *x* < 0; $f^{'}\left(x\right)<0$ for *x* > 0, inc(-∞,0), dec (0,∞)

1. Concavity and Inflection points: $f^{''}(x)=(6x^{2}-2)/(x^{2}+1)^{3}$

CU (-∞,-1/√3) U (1/√3, ∞); CD (-1/√3,1/√3), IP ( ±1/√3, ¾)

1. Graph (label all points)
2. 

4. Sketch the graph of $f\left(x\right)=4x^{3}-x^{4}$

1. Domain: all *x*
2. *x*int =0,4, *y*int =0
3. Symmetry: none
4. VA none; HA none
5. Interval of increase/decrease $f^{'}(x)=12x^{2}-4x^{3}=0$, CP (0,0), (3,27)

$f^{'}\left(x\right)>0$ in (-∞,3); $f^{'}\left(x\right)<0$ in (3,∞), inc(-∞,3), dec (3,∞)

1. Concavity and Inflection points: $f^{''}\left(x\right)=24x-12x^{2}=0, x=0,x=2$

CU (0,2); CD (-∞,0) U (2, ∞); IP ( 0,0), (2, 16)

1. Graph (label all points) Scale: *x*=1, y=3
2. 

**Exercises:**

1. Find the critical points
2. Test for symmetry
3. Find the intervals on which *f* is increasing or decreasing
4. Find the intervals where *f* is concave up or concave down.
5. Find the inflection points if any.
6. Use the information in parts a-e to sketch the graph. Check with a grapher.

1) 

2) 

3) 

4) 

5) 

6) 

7) 

8) 

9) 

10) 

**Answers:**

1. Inc. ; Dec. ; local max ; local min 

 CU (; CD ; IP 

1. Inc. ; Dec. ; local max ;

 local min ; CU ; CD ; IP 

1. Inc. ; Dec. ; local max ;

 local min ; CU ; CD ;

 IP , 

1. Inc. ; Dec. ; local max ; local min 

 CU (; CD ; IP 

1. Inc. ; Dec. ; local max ;

 local min ; CU ; CD ;

 IP 

1. *y*-int :0; *x*-int :0; VA: *x=*1; HA: *y* =1; Dec: ; CU ; CD ; IP: none
2. *y*-int :-1/9; VA: ; HA: ; Inc. ; Dec: ; local max ; CU ; CD ; IP: none
3. *y*-int :1; *x*-int :1; VA: **; HA:; Dec: ; CU ; CD ; IP: none
4. *y*-int :0; *x*-int :0; VA: None; HA: None ; Inc: ; Dec: ; local max ; local min ; CU ; CD ; IP: ;;
5. *y*-int :0; *x*-int :0; Inc: ; Dec: ; local max ; local min ; CU ; CD ; IP: ;;

**Optimization**

 1. A farmer with 800 ft. of fencing wants to enclose a rectangular area and then divide it into four pens with fencing parallel to one side of the rectangle. Use Calculus to find the largest possible total area of the four pens. Show that your area is a maximum. Show all your work.

Ans: A=1600sq ft. ; A”(x)=-5 <0, relative max.

2. A company is constructing an open-top, square-based, rectangular metal tank that will have a volume of 4000 cm3. What dimensions yield the minimum surface area? Use the second derivative to check that your answer gives a minimum. Show all your work.

Ans: x=20, y=10; s”(x) =2+ 32000/x3 |x=20 >0, relative min.

3. Find the point (*x*, *y*) on the curve $y=\sqrt{x }$ that is closest to the point (3, 0). Show that you have a minimum. Show all your work. Ans: (5/2, sqrt(5/2)); (d2)”=2 > 0 relative min.

4. A rectangle is to be inscribed in a circle of radius 2. What is the largest area the rectangle can have, and what are the dimensions. Ans: A=8, L=2√(2), w=2√(2)

5. A landscape architect plans to enclose a 7500 square foot rectangular garden. She will use shrubs costing $20 per foot along three sides and fencing costing $10 per foot along the fourth side. Include a sketch. Find the dimensions of the garden that minimize the cost. Show all the steps. Round your answers to one decimal place.

Ans: 100 by 75.