MAC 2311 Hybrid Calculus I

Sections **4.1-4.2** NO Calculator

**Derivatives and Graphs**

**Critical Points (values)**

If the point (c, *f*(c)) is in the domain of the function$ f$, (c, *f*(c)) is called a critical point (CP) of $f $if $f^{'}\left(c\right)=0$ or $f'(c)$ does not exist (DNE). The number c is called a critical value (CV).

Eg1. Find the CPs of $f\left(x\right)=x^{3}/3-3x^{2}/2+2x+1$

Eg2. Find the CPs $g\left(x\right)=(x-1)^{2/3}$

Eg3. Find the CPs $h\left(x\right)=1/(x+1)$

**Relative (local) Extrema**

If $f$ has a relative or local extrema (relative max/min) at the point (c, *f*(c)), and $f'(c)$ is defined, then$ f^{'}\left(c\right)=0$. The converse is not true. If the derivative is zero, you may or may not may or may not have a relative extrema.

Eg4. $f\left(x\right)=x^{2}+1$

Eg5. $g\left(x\right)=x^{3}$

**Absolute Extrema**

If $f$ is a continuous function on the interval [a, b], the absolute extrema (abs max/min) will occur either at the critical points or at the end points.

Eg6. Find the Absolute Extrema of $f\left(x\right)=x^{2}-1$ [-1, 2]

If $f$ is a continuous function on the interval (a, b), the absolute extreme values (if they exist) will occur at interior points of the interval.

Eg7. Find the Absolute Extrema of $f\left(x\right)=x^{2}-1$ (-1, 2)

**Increasing/decreasing functions**

A continuous function $f $is increasing (inc) on the interval I if $f^{'}\left(x\right)>0$ on I.

A continuous function $f$is decreasing (dec) on the interval I if $f^{'}\left(x\right)<0$ on I.

Eg8. Find the intervals where $f\left(x\right)=x^{3}/3-3x^{2}/2+2x+1$ is increasing/ decreasing

**First Derivative Test (FDT) for Relative Extrema**

Suppose $f$is differentiable at every point on an Interval I that contains the critical value c (except perhaps at c itself).

If $f'$ changes sign from positive to negative as x increases through the value c, then $f$ is a local maximum at c.

If $f'$ changes sign from negative to positive as x increases through the value c, then $f$ is a local minimum at c.

If $f'$ does not change sign through c, then $f$ has no local extrema at c.

Eg9. Use the FDT to find the relative extrema of $f\left(x\right)=x^{5}/5-3x^{4}/4+2x^{3}/3+1$

**Concavity**

From the intuitive stand point, an arc of a curve is said to be concave up if it has the shape of a cup and concave down if it has the shape of a cap.

If $f'^{'}\left(x\right)>0$ on an open interval I, then the graph of $f$ is concave up (CU) on I.

If $f'^{'}\left(x\right)<0$ on an open interval I, then the graph of $f$ is concave down (CD) on I.

**Second Derivative Test (SDT) for Relative Extrema**

Suppose $ f^{''}\left(x\right)$ is continuous on the open interval I containing c with $ f^{'}\left(c\right)=0$ (at CV)

If $ f^{''}\left(c\right)>0$ then $f$ has a local minimum at c.

If $ f^{''}\left(c\right)<0$ then $f$ has a local maximum at c.

If $ f^{''}\left(c\right)=0$ then the test fails. If the test fails, the first derivative test must be used to find local extrema.

Eg10. Use the SDT to find the relative extrema of $f\left(x\right)=x^{5}/5-3x^{4}/4+2x^{3}/3+1$

Notes: If (c, *f*(c)) is a relative extremum of$ f$, then$ f^{'}\left(c\right)=0$.

The converse is not true. If$ f^{'}\left(c\right)=0$, you may or may not may or may not have a relative extremum.

**Test for Concavity (Inflection Points (IP))**

Suppose $ f^{''}\left(x\right)$ exists on the interval I.

If $ f^{''}\left(x\right)>0$ on I, then $f$ is concave up on I.

If $ f^{''}\left(x\right)<0$ on I, then $f$ is concave down on I.

If (c, *f*(c)) is a point on I at which$ f^{''}\left(x\right)=0$, and $ f^{''}\left(x\right)$ changes at c (changes concavity), then (c, *f*(c)) is an inflection point of $f.$

Notes: If (c, *f*(c)) is an inflection point of$ f$, then$ f^{''}\left(c\right)=0$.

The converse is not true. If$ f^{''}\left(c\right)=0$, you may or may not may or may not have an inflection point.

Eg11. Find the inflection points of $f\left(x\right)=x^{4}$

Eg12. Find the inflection points of $g\left(x\right)=x^{4}/4-x^{3}$