MAC 2311 Hybrid Calculus I

Sections **4.1-4.2** NO Calculator

**Derivatives and Graphs**

**Critical Points (values)**

If the point (c, *f*(c)) is in the domain of the function, (c, *f*(c)) is called a critical point (CP) of if or does not exist (DNE). The number c is called a critical value (CV).

Eg1. Find the CPs of

Eg2. Find the CPs

Eg3. Find the CPs

**Relative (local) Extrema**

If has a relative or local extrema (relative max/min) at the point (c, *f*(c)), and is defined, then. The converse is not true. If the derivative is zero, you may or may not may or may not have a relative extrema.

Eg4.

Eg5.

**Absolute Extrema**

If is a continuous function on the interval [a, b], the absolute extrema (abs max/min) will occur either at the critical points or at the end points.

Eg6. Find the Absolute Extrema of [-1, 2]

If is a continuous function on the interval (a, b), the absolute extreme values (if they exist) will occur at interior points of the interval.

Eg7. Find the Absolute Extrema of (-1, 2)

**Increasing/decreasing functions**

A continuous function is increasing (inc) on the interval I if on I.

A continuous function is decreasing (dec) on the interval I if on I.

Eg8. Find the intervals where is increasing/ decreasing

**First Derivative Test (FDT) for Relative Extrema**

Suppose is differentiable at every point on an Interval I that contains the critical value c (except perhaps at c itself).

If changes sign from positive to negative as x increases through the value c, then is a local maximum at c.

If changes sign from negative to positive as x increases through the value c, then is a local minimum at c.

If does not change sign through c, then has no local extrema at c.

Eg9. Use the FDT to find the relative extrema of

**Concavity**

From the intuitive stand point, an arc of a curve is said to be concave up if it has the shape of a cup and concave down if it has the shape of a cap.

If on an open interval I, then the graph of is concave up (CU) on I.

If on an open interval I, then the graph of is concave down (CD) on I.

**Second Derivative Test (SDT) for Relative Extrema**

Suppose is continuous on the open interval I containing c with (at CV)

If then has a local minimum at c.

If then has a local maximum at c.

If then the test fails. If the test fails, the first derivative test must be used to find local extrema.

Eg10. Use the SDT to find the relative extrema of

Notes: If (c, *f*(c)) is a relative extremum of, then.

The converse is not true. If, you may or may not may or may not have a relative extremum.

**Test for Concavity (Inflection Points (IP))**

Suppose exists on the interval I.

If on I, then is concave up on I.

If on I, then is concave down on I.

If (c, *f*(c)) is a point on I at which, and changes at c (changes concavity), then (c, *f*(c)) is an inflection point of

Notes: If (c, *f*(c)) is an inflection point of, then.

The converse is not true. If, you may or may not may or may not have an inflection point.

Eg11. Find the inflection points of

Eg12. Find the inflection points of