**Topics for Test #1 (Chap 1 Sec 1.1-1.3 and Chap2 Sec 2.1-2.6)**

**CHAPTER 1 FUNCTIONS**

The opening chapter of the text focuses on functions: their properties, their graphs, and their use in applications.

It can also be viewed as an overview of the prerequisite knowledge from algebra and trigonometry that is necessary for success in a calculus course.

Additional review material appears in Appendix A.

Since the material of this chapter is basic skills that you must have, you must spend extra time reviewing this material. The main reason for failure in calculus usually stems from difficulties in algebra and trigonometry. Without knowing algebra and trigonometry, you cannot do calculus.

The calculus curriculum starts with Chapter 2.

**1.1 Review of Functions**

**Overview**

A function is defined and its properties are developed.

**Lecture**

Review the definition of a function, its geometric interpretation (the vertical line test), and the concepts of domain and range (both the domain of definition and the domain in the context of an application) Do **Examples1-3**.

Watch the **Video Presentation (Functions)** in the 1.1 Video Lecture link in the Interactive e-book.

See an animation of domain and range at:<http://mathdemos.gcsu.edu/mathdemos/domainrange/domainrange.html>

Watch the video how to find the Domain of a Function at:

<http://video.google.com/videoplay?docid=2378867002563806676#docid=58023305625975124>

Review composition of functions. Composite functions are featured prominently in calculus, and do **Examples 4–7**.

Watch the Video Composition of Functions at <http://www.brightstorm.com/math/algebra-2/functions/composition-of-functions>

Review the notion of symmetry in graphs, and give definitions of even and odd functions, and do **Example 8**.

Watch the Video Even and Odd Functions at <http://www.youtube.com/watch?v=2Tnvwai5cqc>

and at <http://www.youtube.com/watch?v=oKKcIK_PgEk&feature=related>

Watch the **Video Presentation (Even and Odd Functions)** in the 1.1 Video Lecture link in the Interactive e-book.

**1.2 Representing Functions**

**Overview**

We introduce the full catalog of functions that will be encountered in calculus, and present four ways to represent a function: through formulas, graphs, tables, and words.

**Lecture**

Review the standard functions and provide representative graphs for each family of functions. **Examples 1–3**.

Review piecewise functions, which are used repeatedly in the next chapter (limits). Included is the piecewise definition of the absolute value function, another fact that will be used frequently in upcoming material. **Example 4**

Review rational functions and the Area function **Examples 5, 7**

Watch the **Video Presentation (Graphs of Basic Functions)** in the 1.2 Video Lecture link in the Interactive e-book

Watch the Video Piecewise Functions at <http://www.youtube.com/watch?v=-gwffMEr8i8>

You need to review the basic shapes of the standard functions.

Watch the **Video Presentation (Graphing Techniques)** in the 1.2 Video Lecture link in the Interactive e-book

Reviewing transformations of graphs is important, as this topic gives students the tools needed to quickly visualize more complicated functions. **Examples 8-9**

Watch the Video Horizontal and Vertical Graphs Transformation at <http://www.youtube.com/watch?v=3Q5Sy034fok>

**1.3 Trigonometric Functions**

**Overview**

The trigonometric functions are defined, and the graphs and properties of these functions are examined.

**Lecture**

* Review the six trigonometric functions, using both the right-triangle definition and by treating them as circular functions
* You are required to evaluate the trigonometric functions at special angles.

You can either memorize Figure 1.45, or you can evaluate the trigonometric functions by generating the table below for angles in the first quadrant, and with the signs of the trig functions in the other quadrants, you can obtain the other values.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  |  |  |
| **0** |  |  |  |  |
| **30** |  |  |  | **1/** |
| **45** |  |  |  | **1** |
| **60** |  |  |  |  |
| **90** |  |  |  | **Und.** |

The values are obtained when the fractions are simplified. The tangent is obtained by dividing sine by cosine, and the values of the other trigonometric functions are obtained by using the reciprocal relations.

To find the trigonometric functions of non-acute angles:

1. Draw the angle in standard position (from the positive *x*-axis).
2. Find the reference angle (angle with the *x*-axis)
3. Since the reference angle is acute, use the values of the trigonometric functions in the table above with the sign of the trigonometric function in that quadrant.

Eg. Evaluate.

Since  is in quadrant II (cosine negative) with reference angle

of , 

Eg. Evaluate.

Since  is in quadrant IV (tangent negative) with reference angle

of , 

Eg. Evaluate.

Since  is in quadrant III (sin negative) with reference angle

of , 

Eg. Evaluate.

Since  is in quadrant IV (secant positive) with reference angle

 of , 

Quadrantal Angles

To find quadrantal angles, select the point on the terminal side of the angle with distance one from the origin. The coordinate of that point on a unit circle will be

Eg. Evaluate and 

Since the terminal side is in the negative *y*-axis, the point on the unit circle will be: , so  and 

 --- (0,-1)

* Review the trigonometric identities. These identities are the most frequently used in calculus.
* Solve trigonometric equations. See **Example 2**
* Review the graphs of the trigonometric functions.

**CHAPTER 2 LIMITS**

Limits provide the foundation for all the key ideas of calculus (differentiation, integration, and infinite series, to name a few). This chapter supplies the tools that your students will use to understand the important concepts of calculus.

**2.1 The Idea of Limits**

**Overview**

In this opening section, we introduce the idea of a limit through an investigation of the relationship between instantaneous velocity and tangent lines. The intent is to provide the motivation for the entire chapter and to give an intuitive sense of how limits work.

**Lecture**

 Limits arise naturally when we define instantaneous velocity and the line tangent to a curve.

Click on the **Average Velocity** link in your Interactive e-book (see **Example 1)**. To see the solution of Example 1, click on solution.

We can see that average velocities are just slopes of secant lines on a position curve (Figures 2.1–2.3).

Click on the **Instantaneous Velocity** link (see **Example 2)**. Notice that average velocity can be used to approximate instantaneous velocity. We can see that shrinking the time interval leads to better approximations of instantaneous velocity. We introduce the idea of a limit in moving from average to instantaneous velocity.

Click on the **Slope of Tangent Lines** link**.**

In this paragraph, we introduce the notion of a tangent line as the limit of approaching secant lines.

**2.2 Definitions of Limits**

**Overview**

This section gives a standard treatment of left-hand, right-hand, and two-sided limits, and shows how to compute them (informally) with graphs and numerical methods.

**Lecture**

Read the preliminary (informal) definition of a limit.

Computing the value of a limit is most easily carried out with a graph (see **Example 1**).

Notice that the limit of a function *f* at the point *a* (if it exists) does not depend upon the value of *f* (*a*).

The value of a limit at a point can also be investigated by tabulating function values near that point (see **Example 2**).

Read the definition of one sided limits, and watch the **Video Presentation (One Sided Limits)** in the 2.2 Video Lecture link in the Interactive e-book

**Example 3** shows the left-and right-hand limits and their relationship to the corresponding two-sided limit.

The limits may fail to exist, either because the left- and right-hand limits do not agree (see **Example 4**), or because the values of a function do not approach a single number (**Example 5**).

**2.3 Techniques for Computing Limits**

**Overview**

Analytical methods for evaluating limits are presented.

**Lecture**

Limits of polynomial, rational, and algebraic functions can usually be evaluated by direct substitution, provided the function is defined at the limit point (exceptions include piecewise functions; see Example 5).

Limit laws and algebraic manipulation provide tools for evaluating more challenging limits.

Start by studying Theorem 2.3 (Limit laws), and apply them to **Example2.**

Now, watch the Video Presentation (Calculating Limits Using the Limit Laws) in the 2.3 Video Lecture link in the Interactive e-book.

As you saw in the video, limits that cannot be evaluated by direct substitution can often be transformed into limits that yield to direct substitution via factoring and multiplication by the conjugate.

For additional examples, see **Examples 3 though 6**

Example 7 uses the Squeeze Theorem to help you find some limits.

**2.4 Infinite Limits**

**Overview**

Infinite limits are introduced (initially alongside a limit at infinity to help students distinguish between the two scenarios), and their connection to vertical asymptotes is explained.

**Lecture**

Start reading ‘An Overview’ to distinguish betweenInfinite Limits and Limit at Infinity.

It is important that you understand that when an infinite limit does not exist, we use the symbol ∞ (or −∞) as a convenience to indicate that the function attains arbitrarily large values. See the definition: Infinite Limits.

We say a two-sided limit is ∞ (or −∞) only when the left- and right-hand limits “agree,” despite the fact that neither the left nor right-hand limit exists (see **Example 1**).

Remember that the limits may fail to exist, either because the left- and right-hand limits do not agree (see **Example 4 in section 2.2**), because the values of a function do not approach a single number (**Example 5 in section 2.2**), or because the limit is ∞ (or −∞) .

The graph of *f* has the vertical asymptote *x* = *a* whenever the limit of *f* (left, right, or two-sided) is infinite in magnitude at *a*. See **Example 2**

Now, watch the Video Presentation (Infinite Limits and Asymptotes) in the 2.4 Video Lecture link in the Interactive e-book.

For additional examples, see **Examples 3 though 5**

**2.5 Limits at Infinity**

**Overview**

Limits at infinity determine the end behavior of a function, detect the presence of horizontal asymptotes, and reveal whether a system attains a steady state.

**Lecture**

When they exist, limits at infinity indicate a horizontal asymptote. See the definition of Limits at Infinity and Horizontal Asymptote.

**Example1** shows you how to compute a simple limit.

Infinite limits at infinity do not exist because the limit is ∞ (or −∞) (see section 2.4), whereas $\lim\_{x\to \infty }cos⁡(x)$ because the limit does not give a number (fluctuate between -1 and 1)

Review Theorem 2.6, and do **Example 2** for some infinite limits at infinity. **Example 3** deals with the end behavior of rational functions.

Now, watch the Video Presentation (Limits at Infinity) in the 2.5 Video Lecture link in the Interactive e-book.

Theorem 2.7 helps you work limits in **Example 4**.

Infinite limits at infinity do not exist because the limit is ∞ (or −∞) (see section 2.4), whereas $\lim\_{x\to \infty }sin(x)$ and $\lim\_{x\to \infty }cos⁡(x)$ do not exist because the limit does not give a number (oscillate between -1 and 1)

**2.6 Continuity**

**Overview**

A standard treatment of continuity is offered, with the important Intermediate Value Theorem given at the end of the section.

**Lecture**

We are going to avoid defining the phrase “continuous function,” which is usually taken to mean a function continuous on its domain. Rather, we are careful to claim that a function is continuous either at a point, or on an interval (occasionally specified only as the domain of the function in question, which of course could be a collection of intervals). The reason behind this decision:

It is correct to say that *f* (*x*) = 1/ *x* is a continuous function, and yet it has a discontinuity at *x* = 0. Avoiding this apparent inconsistency in terminology is easier when you are encountering continuity for the first time.

Now, watch the Video Presentation (Continuity) in the 2.6 Video Lecture link in the Interactive e-book.

Read the definition of Continuity at a Point. The definition of continuity allows the use of direct substitution when evaluating$ \lim\_{x\to a}f(x)$, provided *f* is continuous at *a*.

It is important that you read the Continuity Checklist, because for continuity at a point all 3 conditions must hold.

Do **Example 1** to learn how to distinguish the different types of discontinuities.

As you will see, the standard families of functions (polynomial, rational, trigonometric, etc.) are continuous on their domains. Do **Examples 2 and 3.**

Theorem 2.10 is used to evaluate limits of composition of functions. **See Example 4**

Study the definition of Continuity at Endpoints and Continuity on an Interval along with **Example 5**. StudyFunctions Involving Roots with **Example 6.** Read The Intermediate Value Theorem with **Example 8.**

**Chapter 2 Key Terms and Concepts**

Average velocity (Section 2.1)

Instantaneous velocity (Section 2.1)

Slope of the tangent line (Section 2.1)

Informal limit definitions (Section 2.2)

Relationship between one-sided and two-sided limits (Theorem 2.1) (Section 2.2)

Limits of linear functions (Theorem 2.2) (Section 2.3)

Limit laws (Theorem 2.3) (Section 2.3)

Limits of polynomials and rational functions (Theorem 2.4) (Section 2.3)

Squeeze Theorem (Theorem 2.5) (Section 2.3)

Definition of various infinite limits (Section 2.4)

Vertical asymptotes (Section 2.4)

Definition of various limits at infinity (Section 2.5)

End behavior of polynomials (Theorem 2.6) (Section 2.5)

End behavior of rational functions (Theorem 2.7) (Section 2.5)

Continuity at a point (Section 2.6)

Continuity check list (Section 2.6)

Continuity of sums, products, quotients (Theorem 2.8) (Section 2.6)

Common continuous functions (Theorem 2.9) (Section 2.6)

Continuity of composite functions (Theorem 2.10) (Section 2.6)

Left-continuous and right-continuous functions (Section 2.6)

Continuity on an interval (Section 2.6)

Continuity of functions with roots (Theorem 2.11) (Section 2.6)

Intermediate Value Theorem (Theorem 2.13) (Section 2.6)