LINEAR AND ANGULAR VELOCITY

Consider the formula for arc length of a sector of a circle $s = r\theta$.

If we divide by time *t*, we obtain $\frac{s}{t} = r\frac{\theta}{t}$. This can be written as $V = r\omega$ where $V = \frac{s}{t}$ is the linear velocity of a point on a circle or the change in arc length per unit of time, and $\omega = \frac{\theta}{t}$ is the angular velocity of a point on a circle or the change in radian measure per unit of time.

Example1: Suppose that point *P* is on a circle with radius 10 cm, and ray *OP* is rotating with angular velocity $\pi/18$ radian per second.

(a) Find the angle generated by *P* in 6 seconds.

Ans: $\theta = \pi/3$ rad.

(b) Find the distance traveled by *P* along the circle in 6 seconds

Ans: $s = 10\pi/3$ cm.

(c) Find the linear velocity of *P*

Ans: $v = 5\pi/9$ cm per second.

Example 2: A belt runs a pulley of radius 6 cm at 80 revolutions per minute.

(a) Find the angular velocity of the pulley in radians per second.

Ans: $\omega = 8\pi/3$ rad/s

(b) Find the linear velocity of the belt in centimeters per second.

$v = 16\pi$ cm/s.

Example 3: A satellite traveling in a circular orbit 1600 km above the surface of Earth takes two hours to make an orbit. Assume that the radius of Earth is 6400 km.

(a) Find the linear velocity of the satellite.

Ans: $v = 8000\pi$ km/ hr.

(b) Find the distance traveled in 4.5 hours.

Ans: $36,000\pi$ km.

Exercises:

1. If a point moves around the circumference of a unit circle at an angular velocity of 1 radian per second, how long will it take for the point to move around the entire circle?

Ans: 2π sec.

2. If a point moves around the circumference of a unit circle at a speed of 2 units per second, how long will it take for the point to move around the entire circle? Ans: π sec

3. Find ω for each of the following.

a) The minute hand of a clock b) the second hand of a clock Ans: $\pi/30$rad/min $\pi/30$ rad/sec.

4. Find *v* for the tip of the hour hand of a clock, if the hand is 6cm long. Ans: π cm/hr

5. Find *v* for the tip of an airplane propeller blade 2 m long, rotating 500 times per minute.

Ans: 2000 π m/min.

6. The tires of a bicycle have radius 13 inches and are turning at the rate of 200 revolutions per minute. How fast is the bicycle traveling in miles per hour? *(Hint:* 5280 feet = 1 mile.)
Ans: 15.5mi/h

7. Mars rotates on its axis at the rate of .2552 radians per hour. Approximately how many hours are in a Martian day? Ans: 24.6206hrs.

8. Earth travels about the sun in an orbit that is almost circular. Assume that the orbit is a circle, with radius 93,000,000 miles. Its angular and linear velocities are used in designing solar power facilities.
(a) Assume that a year is 365 days, and find θ, the angle formed by Earth's movement in one day. Ans: $\pi/182.5$rad
(b) Give the angular velocity in radians per hour. Ans: $\pi/4380$ rad/hr.
(c) Find the linear velocity of Earth in miles per hour. Ans: 66,700 mi/hr.

9. The two pulleys connected by a belt have radii of 15 cm and 8 cm, respectively. The larger pulley rotates 25 times in 36 seconds. Find the angular velocity of each pulley in radians per second. Ans: large: $25\pi/18$; small: $125\pi/48$

10. A thread is being pulled off a spool at the rate of 59.4 cm per second. Find the radius of the spool if it makes 152 revolutions per minute. Ans: 3.73cm.

11. Earth revolves on its axis once every 24 hours. Assuming that Earth's radius is 6400 km. find the following:
(a) angular velocity of Earth in radians per day and radians per hour. Ans: $\pi/12$rad/hr
(h) linear velocity at the North Pole or South Pole. Ans: $v = 0$
(c) linear velocity at Quito, Ecuador, a city on the equator Ans: $13\pi/3$ km/hr.
(d) linear velocity at Salem, Oregon (halfway from the equator to the North Pole)
Ans: $13\sqrt{2}\pi/6$ km/hr.

12. A railroad track is laid along the arc of a circle of radius 1800 feet. The circular part of the track subtends a central angle of 40°. How long (in seconds) will it take a point on the front of a train traveling 30 mph to go around this portion of the track? Ans: 28.56 s.

13. A 90-horsepower outboard motor at full throttle will rotate its propeller at 6000 revolutions per minute. Find the angular velocity of the propeller in radians per second.
Ans: 200π rad/s.
14. An electrical wind generator has propeller blades that are 5.00 m long. If the blades are rotating at 8π rad/sec, what is the linear velocity (to the nearest meter per second) of a point on the tip of one of the blades?
Ans: $v = 126$ m/s
15. A point on the rim of a 6.0 in. diameter wheel is traveling at 75 ft/sec. What is the angular velocity of the wheel (in radians per second)?
Ans: $\omega = 300$ rad/s

16. If a 6 cm shaft is rotating at 4,000 rpm (revolutions per minute), what is the speed of a particle on its surface (in centimeters per minute, to two significant digits)?
Ans: $v = 24,000\pi$ cm/min

17. The 25,000 lb Hubble space telescope was launched April 1990 and placed in a 380 mi circular orbit above the earth's surface. It completes one orbit every 97 min, going from a dawn-to-dusk cycle nearly 15 times a day. If the radius of the earth is 3,964 mi, what is the linear velocity of the space telescope in miles per hour (mph)?
Ans: 17,000 mph.

18. A 16 mm diameter shaft rotates at 1,500rps (revolutions per second). Find the speed of a particle on its surface (to the nearest meter per second).

19. A 6 cm diameter shaft rotates at 500rps. Find the speed of a particle on its surface (to the nearest meter per second).

20. An earth satellite travels in a circular orbit at 20,000 mph. If the radius of the orbit is 4,300 mi, what angular velocity (in radians per hour, to three significant digits) is generated?

21. A bicycle is ridden at a speed of 7.0 m/sec. If the wheel diameter is 64 cm, what is the angular velocity of the wheel in radians per second?

22. The velocity of sound in air is approx. 335.3 m/sec. If an airplane has a 3.000 m diameter propeller, at what angular velocity will its tip pass through the sound harrier?

23. If an electron in an atom travels around the nucleus in a circular orbit at 8.11×10^{6} cm/s, what angular velocity (in radians per second) does it generate, assuming the radius of the orbit is 5.00×10^{-9} cm?