

Identities

$$\sin^2(a) + \cos^2(a) = 1, \sec^2(a) - \tan^2(a) = 1, \csc^2(a) - \cot^2(a) = 1$$

$$\cos(a \mp b) = \cos(a)\cos(b) \pm \sin(a)\sin(b), \sin(a \pm b) = \sin(a)\cos(b) \pm \cos(a)\sin(b)$$

$$\sin(2a) = 2\sin(a)\cos(a), \cos(2a) = \begin{cases} \cos^2(a) - \sin^2(a) \\ 1 - 2\sin^2(a) \\ 2\cos^2(a) - 1 \end{cases}, \tan(2a) = \frac{2\tan(a)}{1 - \tan^2(a)}$$

$$\sin^2 x = \frac{1 - \cos 2x}{2}, \cos^2 x = \frac{1 + \cos 2x}{2}, \sin\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1 - \cos x}{2}}, \cos\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1 + \cos x}{2}}$$

Inverse Functions

$$y = \sin^{-1} x, -\pi/2 \leq y \leq \pi/2, \quad y = \csc^{-1} x = \sin^{-1} 1/x, -\pi/2 < y < \pi/2, y \neq 0$$

$$y = \cos^{-1} x, 0 \leq y \leq \pi, \quad y = \sec^{-1} x = \cos^{-1} 1/x, 0 < y < \pi, y \neq \pi/2$$

$$y = \tan^{-1} x, -\pi/2 < y < \pi/2, \quad y = \cot^{-1} x = \frac{\pi}{2} - \tan^{-1} x, 0 < y < \pi$$

The Inverse functions formulas show the way the functions are restricted.

Examples:

Find $\sin^{-1} 1/2$. This problem asks for the angle whose sine is $1/2$. Since the sine is restricted from $-\pi/2 \leq x \leq \pi/2$ and the sine is positive in the first quadrant, $y = \sin^{-1} 1/2 = \pi/6$ is the angle desired.

Find $\sin^{-1} -\sqrt{3}/2$. This problem asks for the angle whose sine is $-\sqrt{3}/2$. Since the sine is restricted from $-\pi/2 \leq x \leq \pi/2$ and the sine is negative in the fourth quadrant, $\sin^{-1} -\sqrt{3}/2 = -\pi/3$ is the angle desired.

Find $\cos^{-1} 1/2$. This problem asks for the angle whose cosine is $1/2$. Since the cosine is restricted from $0 \leq x \leq \pi$ and the cosine is positive in the first quadrant, $\cos^{-1} 1/2 = \pi/3$ is the angle desired.

Find $\cos^{-1} -\sqrt{3}/2$. This problem asks for the angle whose cosine is $-\sqrt{3}/2$. Since the cosine is restricted from $0 \leq x \leq \pi$ and the cosine is negative in the second quadrant, $y = \cos^{-1} -\sqrt{3}/2 = 5\pi/6$ is the angle desired. Note that $5\pi/6$ has a reference angle of $\pi/6$.

Find $\cot^{-1} -\sqrt{3}$. To find the angle whose cotangent is $-\sqrt{3}$, we use the formula $\cot^{-1} -\sqrt{3} = \frac{\pi}{2} - \tan^{-1} -\sqrt{3}$.

Since the tangent is restricted from $-\pi/2 < x < \pi/2$ and the tangent is negative in the fourth quadrant,

$$\tan^{-1} -\sqrt{3} = \frac{\pi}{3}, \text{ so } \cot^{-1} -\sqrt{3} = \frac{\pi}{2} - \tan^{-1} -\sqrt{3} = \frac{\pi}{2} - \left(-\frac{\pi}{3}\right) = \frac{5\pi}{6}$$

Find $\sec^{-1} -2$. To find the angle whose secant is -2, we use the formula $y = \sec^{-1} -2 = \cos^{-1} -1/2 = 2\pi/3$, since the cosine is negative in the second quadrant.

Find $\csc^{-1} -2$. To find the angle whose cosecant is -2, we use the formula $y = \csc^{-1} -2 = \sin^{-1} -1/2 = -\pi/6$, since the sin is negative in the fourth quadrant.