Algebraic Addition of Sinusoids With the Same Frequency

The sum of two sinusoids with the same frequency of the form $y = Ncos(bt) + Msin(bt)$, where N and M are real numbers, can be expressed as $y = Asin(bt + c)$ where $A = \sqrt{N^2 + M^2}$ and $tan(c) = \frac{N}{M}$.

<u>Proof:</u>

Given $y = Ncos(bt) + Msin(bt)$, if we let $N = Asin(c)$, and $M = Acos(c)$ the expression becomes $y = Asin(c)cos(bt) + Acos(c)sin(bt)$= $Asin(bt + c)$ (sine of sum of angles formula).
If we add the squares of N and M we obtain $N^2 + N^2 = A^2$ or $A = \sqrt{N^2 + M^2}$.
If we divide N by M we obtain $tan(c) = \frac{N}{M}$ where the quadrant of c will be given by the signs of N and M.
We can find the angle c by the signs of N and M .
Since we are assuming $A > 0$, the sign of N is the same as sign of $sin(c)$ ($N = Asin(c)$) and the sign of M is the same as the sign of $cos(c)$($M = Acos(c)$). Lets say $N > 0$ ($sin(c) > 0$)and $M < 0$ ($cos(c) < 0$), c is in the second quadrant. The quadrants for the other three combinations of the signs of N and M can be found the same way.

eg. Express $y = 2cos(3x) + 2\sqrt{3}sin(3x)$ as $Asin(bx + c)$.
$A = \sqrt{2^2 + \left(2\sqrt{3}\right)^2} = 4$, $c = tan^{-1}(2/2\sqrt{3}) = tan^{-1}(1/\sqrt{3}) = \pi/6$.
Since N and M are positive, the angle is in the first quadrant, so the form is $y = 4sin(3x + \pi/6)$

eg. Express $y = -cos(2x) - \sqrt{3}sin(2x)$ as $Asin(bx + c)$.
$A = \sqrt{1^2 + \sqrt{3}^2} = 2$, $c = tan^{-1}(-1/-\sqrt{3}) = tan^{-1}(1/\sqrt{3}) = \pi/6$.
Since N and M are negative, the angle is in the third quadrant, so the form is $y = 2sin(2x + 7\pi/6)$.

eg. Express $y = 3\,cos(2x/5) - 3sin(2x/5)$ as $Asin(bx + c)$.
$A = \sqrt{3^2 + 3^2} = 3\sqrt{2}$, $c = tan^{-1}(3/-3) = tan^{-1}(-1) = -\pi/4$. Since $N = Asin(c)$ is positive and $M = Acos(c)$ is negative, the angle is in the second quadrant, so the form is $y = 3\sqrt{2}sin(2x/5 + 3\pi/4)$