*Review 6.1 (****Key****)*

1. Identify the exponential function(s).

 a.$ f(x$) = $3^{x}$ **Yes**  b. $f(x$) = $x^{3}$ **No** c. $f(x$) = $(-7)^{x}$ **No**  d. $f(x$) = $-7^{x}$ **Yes**

**Note:** $f(x$**) =** $-7^{x}$ **is not the same as** $f(x$**) =** $(-7)^{x}$**. The exponential function** $f(x$**) =** $-7^{x}$ **is equivalent to multiplying** $f(x$**) =** $7^{x}$ **by** $-1$**, whereas** $f(x$**) =** $(-7)^{x}$ **would not be an exponential function because it would have a negative base, *b* =** $-7$**.**

2. Graphs of exponential functions:

 a.

|  |  |
| --- | --- |
| ***x*** | ***y*** |
| **0** | $$3^{0}=1$$ |
| **1** | $$3^{1}=3$$ |
| **2** | $$3^{2}=9$$ |
| $-$**1** | $3^{-1}=$ **1/3** |
| $-$**2** | $3^{-2}=$ **1/9** |

 

 b.

|  |  |
| --- | --- |
| ***x*** | ***y*** |
| **0** | $(1/3)^{0}$ **= 1** |
| **1** | $(1/3)^{1}$**= 1/3** |
| **2** | $(1/3)^{2}$**= 1/9** |
| $-$**1** | $(1/3)^{-1} $**= 3** |
| $-$**2** | $(1/3)^{-2} $**= 9** |

 

3. Does $f(x$) = $b^{x}$ have an inverse function?

 **Yes, it is one-to-one; passes the horizontal line test**

4. Use the calculator to graph $f(x$) = $5(3)^{x}$ and $f(x$) = $\frac{1}{5}(3)^{x}$; compare each graph with that of $f(x$) = $3^{x}$. Find the vertical intercept for each graph.

  

 **[-5, 5, 1] by [-2, 8, 1] [-5, 5, 1] by [-2, 8, 1]**

**Note: On each screen shot, dotted graph is the graph of** $f(x$**) =** $3^{x}$**; vertical intercept (0, 1)**

$f(x$**) =** $5(3)^{x}$**: graph is stretched by a factor of 5; vertical intercept (0, 5)**

$f(x$**) =** $\frac{1}{5}(3)^{x}$**: graph is compressed by a factor of 1/5; vertical intercept (0, 1/5) or (0, 0.2)**

5. Compare graphs of $f(x$) = $2^{x}$ and $f(x$) = $-2^{x}$.

$f(x$**) =** $-2^{x}$ **is equivalent to** $f(x$**) =** $-1(2)^{x}$**; reflection about the** $x$**-axis.**

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 ***Reminder*:** $f(x$**) =** $-2^{x}$ **≠** $ f(x$**) =** $(-2)^{x}$

$ f(x$**) =** $(-2)^{x}$ **would not be an exponential function, since it has a negative base.**

6. For each function, do the following without graphing: Find the vertical intercept and state whether it is an increasing or decreasing function.

 a. $f(x)$ = $5(3.2)^{x}$

  **Vertical intercept: (0, 5)** $b =$ **3.2 Since 3.2 > 1 increasing**

 b. $f(t)$ = $\frac{1}{4}\left(\frac{8}{3}\right)^{t}$

 **Vertical intercept: (0, 1/4)** $b =$ **8/3 Since 8/3 > 1 increasing**

 c. $y$ = $0.8(0.7)^{x}$

 **Vertical intercept: (0, 0.8)** $b =$ **0.7 Since 0.7 < 1 decreasing**

 d. $f(x)$ = $8^{x}$= $1(8)^{x}$

 **Vertical intercept: (0, 1)** $b =$ **8 Since 8 > 1 increasing**

 e. $f(x)$ = $\left(\frac{4}{3}\right)^{-x}$= $\left(\frac{3}{4}\right)^{x}$=$1\left(\frac{3}{4}\right)^{x}$

 **Vertical intercept: (0, 1)** $b =$ **3/4 Since 3/4 < 1 decreasing**

7. State any transformations on each exponential function.

 a. $f(x$) = $3^{x}+ 5$ **Graph of** $f(x$**) =** $3^{x}$ **has been shifted 5 units up**



 b. $f(x$) = $3^{x + 5}$ **Graph of** $f(x$**) =** $3^{x}$ **has been shifted 5 units left**

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 c. $f(x$) = $3^{x- 6}-2$ **Graph of** $f(x$**) =** $3^{x}$ **has been shifted 6 units right, 2 units down**

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8. For each function, find the initial value and the growth or decay factor.

 a. $P(t)$ = $2.4(5.6)^{t}$ $P\_{0}$ **= 2.4 growth factor = 5.6**

 b. $N\left(x\right)$ = $21.3(0.8)^{x}$ $N\_{0}$ **= 21.3 decay factor = 0.8**

9. The number of Facebook active users in millions can be modeled by the function

*P*(*t*) *=* $1.53(2.82)^{t}$, where *t* is the number of years after 2004.

 a. Identify and interpret $P\_{0}$.

 $P\_{0}$ **= initial value, that is, population at time = 0.**

$P\_{0}$ **= 1.53 Facebook had ≈ 1.53 million active users in 2004.**

 b. Identify the growth or decay factor.

 **Growth factor = 2.82**

 c. Without graphing, determine whether the function is increasing or decreasing. Explain your

 decision.

$b =$ **2.82 Since 2.82 > 1 increasing**

 d. Approximate the number of Facebook active users in 2008.

 **We know *t* represents the number of years after 2004.**

 **In 2008,** $t$ **≈ 4. Therefore,** $P(4)=$$1.53(2.82)^{4}$ **≈ 96.756 ≈ 97 million active users**

 e. Find the average rate of change in the number of Facebook active users between 2004-2008.

 Round your answer to the nearest whole number.

 **For 2004,** $t$ **= 0, and for 2008,**$ t$ **= 4.**

$P(0)=$$1.53(2.82)^{0}$$=1.53 \left(1\right)= 1.53$ **and** $P(4)=$$1.53(2.82)^{4}$ **≈ 97**

 **So, we have the points** $(0, 1.53)$ **and** $(4, 97)$**.**

$\frac{∆y}{∆x}= \frac{97-1.53}{4 - 0}$ **≈ 24 million active users**

10. For each of the following, identify the growth or decay factor; state the percent increase or decrease.

 a. *P*(*t*) *=* $5(1.25)^{t}$

 **Growth factor 1.25 ⇒ 1 + .25 ⇒ increase of 25%**

 b. *N*(*t*) *=* $210,000(1.015)^{t}$

 **Growth factor 1.015 ⇒ 1 + .015 ⇒ increase of 1.5%**

 c. *P*(*t*) *=* $8,700(0.98)^{t}$

 **Decay factor 0.98 ⇒ 1** $- $**.98 ⇒ decrease of 2%**

 d. *V*(*t*) *=* $35,000(0.655)^{t}$

 **Decay factor 0.655 ⇒ 1** $-$ **.655 ⇒ decrease of 34.5%**