Help with Difference Quotient Formula

Problem 1: Exercise with no variable *x* on the second term

Calculate the difference quotient for $f(x) = 5x^2 - 2$.

Difference Quotient formula: $\dfrac{f(x+h) - f(x)}{h}$

Before you apply the formula, calculate $f(x+h)$, that is, replace "*x*" in "$5x^2 - 2$" with "*x* + *h*"

$f(x+h) = 5(x+h)^2 - 2$

Caution! $(x+h)^2 \neq x^2 + h^2 \longrightarrow (x+h)^2 = (x+h)(x+h) = x^2 + 2xh + h^2$

Therefore,

$$f(x+h) = 5(x+h)^2 - 2 = 5(x^2 + 2xh + h^2) - 2 = 5x^2 + 10xh + 5h^2 - 2$$

Now apply the formula, replacing $f(x+h)$ with: $5x^2 + 10xh + 5h^2 - 2$

$$\frac{f(x+h) - f(x)}{h} = \frac{(5x^2 + 10xh + 5h^2 - 2) - (5x^2 - 2)}{h}$$

Simplifying, we have:

$$= \frac{5x^2 + 10xh + 5h^2 - 2 - 5x^2 + 2}{h}$$

$$= \frac{10xh + 5h^2}{h}$$

$$= \frac{(h)(10x + 5h)}{h} \quad = 10x + 5h \quad \textbf{or} \quad 5(2x + h) \quad \text{both answers are acceptable}$$

Problem 2: Exercise with a variable *x* on the second term

Calculate the difference quotient for $f(x) = 3x^2 - x$.

Difference Quotient formula: $\dfrac{f(x+h) - f(x)}{h}$

Before you apply the formula, calculate $f(x+h)$: replace every "*x*" in "$3x^2 - x$" with "*x* + *h*"

$f(x+h) = 3(x+h)^2 - (x+h)$

Caution! $(x+h)^2 \neq x^2 + h^2 \longrightarrow (x+h)^2 = (x+h)(x+h) = x^2 + 2xh + h^2$

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Therefore,

$$f(x+h) = 3(x+h)^2 - (x+h) = 3(x^2 + 2xh + h^2) - (x+h)$$

$$= 3x^2 + 6xh + 3h^2 - x - h$$

Now apply the formula, replacing $f(x+h)$ with: $3x^2 + 6xh + 3h^2 - x - h$

$$\frac{\boldsymbol{f(x+h) - f(x)}}{\boldsymbol{h}} = \frac{(3x^2 + 6xh + 3h^2 - x - h) - (3x^2 - x)}{h}$$

Simplifying, we have:

$$= \frac{3x^2 + 6xh + 3h^2 - x - h - 3x^2 + x}{h}$$

$$= \frac{6xh + 3h^2 - h}{h}$$

$$= \frac{(h)(6x + 3h - 1)}{h}$$

$$= 6x + 3h - 1$$