MHF 2300

Exam 1

① a) Let a, b be integers with $a \neq 0$, then a divides b means there exists an integer q such that $b = aq$. (2)

b) Let x be a real number, then x is rational provided there exists integers p and q with $q \neq 0$ such that $x = \frac{p}{q}$. (2)

c) Let c, d be integers and n a natural number, then c is congruent to d modulo n means n divides $c - d$. (2)

② a) Let s, t be integers with $t > 0$. The Division Algorithm states there exist unique integers q and r such that $s = tq + r$ and $0 \leq r < t$. (2)

b) Let $s = 70$ and $t = 6$ then $70 = 6(11) + 4$ and $0 \leq 4 < 6$. (1)

c) Let $s = -11$ and $t = 20$ then $-11 = 20(-1) + 9$ and $0 \leq 9 < 20$. (1)

③ a) $x \geq 0$ and $x \neq 5$ (2)

b) n is an even integer or n is a multiple of 5. (2)

④ a) If n is odd then n^2 is not a multiple of 2. (2)

b) If n is even then n^2 is a multiple of 2. (2)

c) The contrapositive is logically equivalent to the original proposition. (1)

⑤ Let f be a real valued function with domain D, then f is not bounded (4) provided that for any real number M there exists $x \in D$ such that $|f(x)| > M$.

⑥ Suppose our universe is the set of real numbers. The statement $\forall x \, \exists y \; x + y = y$ is False. (4)

(7)

p	q	$\sim p$	$\to$	q	$p \vee q$
T	T	F	T	T	T
T	F	F	T	F	T
F	T	T	T	T	T
F	F	T	F	F	F

same

so $\sim p \to q \equiv p \vee q$ (3)

(8) If 3 divides bc and 3 does not divide b then 3 divides c. (3)

(9) a) 10, 17, 24, 31, 38 (1)

b) $543 \equiv 4 \pmod{11}$ (1)

c) If $m \equiv 5 \pmod 8$ and $n \equiv 2 \pmod 8$ then $mn \equiv 2 \pmod 8$ (1)

d) $n \equiv 3 \pmod 8$ (2)

(10) Apply the Division Algorithm to 3 and n, then $n = 3q + r$ with $0 \le r < 3$ for some integers q and r. The hypothesis of the theorem rules out the possibility for $r = 0$ so the only cases left to consider would be $n = 3q + 1$ or $n = 3q + 2$. (2)

(11) Set $a = 4$, $b = 6$, $c = 10$. Now $4 \mid (6 \cdot 10)$ because $6 \cdot 10 = 4(15)$ but $4 \nmid 6$ and $4 \nmid 10$. (4)
This makes the hypothesis of the conjecture true and the conclusion false, so the conjecture is false.

(12) Let a, b, c be integers with $a \neq 0$ and $b \neq 0$. Assume that $a \mid b$ and $b \mid c$. This means (6) there exists integers m and n such that $\underset{(1)}{b = an}$ and $\underset{(2)}{c = bm}$. If we substitute eqn (1) into eqn (2) we obtain $c = (an)m$ and by associativity $c = a(n \cdot m)$ since integers are closed under multiplication we know that $n \cdot m$ is an integer and the defn. of divides tells us that a divides c. The integer we are required to find for this proof is $q = n \cdot m$.