TOTAL= 38

Proof and Logic Exam 2

Don't put work or answers on the test. Write clearly and answer questions completely.

1. Suppose that the universal set is $\{1,2,3,4,5,6,7,8,9,10\}$. Let $A = \{1,2,4,5\}$, $B = \{2,3,5,6\}$, $C = \{2, 6,7,8,9\}$. List the elements in each of the following sets.

A) $A \cup (B \cap C)$

B) $A - B$

C) $(A \cap C)^c$

2. Suppose $A = \{a,b,c,d\}$ and $B = \{p,q,r,s\}$.

A) Give an example of a function $f: A \to B$ which is an injection.

B) Give an example of a function $g: A \to B$ which is not a surjection.

3. The operations $\oplus$ and $\otimes$ are addition and multiplication on Z_8. Calculate the indicated values.

A) $[4] \oplus [5] =$ _____

B) $[3] \otimes [5] =$ _____

C) $[3]^3 =$ _____

D) Find a and b such that $[a] \neq [0]$, $[b] \neq [0]$ but $[a] \otimes [b] = [0]$

4. Let Z be the set of integers and let R be the set of real numbers, and the Z×R is the Cartesian product. Classify each of the following as True or False.

A) $13 \in Z \times R$

B) $\pi \in Z \times R$

C) $(10,\pi) \in Z \times R$

D) $(\pi,10) \in Z \times R$

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5. Suppose that A and B are subsets of a universal set. Fill in the blank with the either the word "and" or the word "or". Don't put your answers on the test.

A) $x \in A - B$ means that $x \in A$ ____ $x \notin B$.

B) $x \notin A \cup B$ means that $x \notin A$ ____ $x \notin B$.

C) $x \notin A \cap B$ means that $x \notin A$ ____ $x \notin B$.

6. Let N be the set of natural numbers. Suppose $d : N \rightarrow N$ where d(n) is the number of natural number divisors of n. For example d(6) = 4 since 1,2,3,6 are the natural number divisors of 6.

A) Find d(15).

B) Is d(15) = d(3)*d(5) ?

C) Is d(8) = d(2)*d(4) ?

D) Is d an injection ? Explain.

6 7. Prove DeMorgan's Law: $(A \cup B)^c = A^c \cap B^c$.

6 8. Suppose $f : R \rightarrow R$ is defined by $f(x) = 3x + 5$. Prove that f is a bijection.

6 9. Use mathematical induction to prove: For all $n \in N$

$$3+6+9+\ldots 3n = \frac{3n(n+1)}{2}$$