MHF 2300 Logic and Proof
Exam 1

Directions: Answer all questions clearly and completely on a separate sheet of paper.

1. Write precise definitions for each of the following terms.

    a) Define a divides b where a, b are integers with $a \neq 0$.

    b) Define x is rational where x is a real number.

    c) Define c is congruent to d modulo n where c, d are integers and n is a natural number.

2. Suppose s and t are integers with $t > 0$.

    a) State the Division Algorithm precisely in terms of s and t.

    b) Give an example of the Division Algorithm (as stated in part a) where $s = 70$ and $t = 6$.

    c) Give an example of the Division Algorithm (as stated in part a) where $s = -11$ and $t = 20$.

3. Write the negation of each of the following predicates. Don't worry about quantifiers here.

    a) $x < 0$ or $x = 5$

    b) n is an odd integer and n is not a multiple of 5.

4. Proposition: If n^2 is a multiple of 2 then n is even.

    a) Write the contra positive of the proposition.

    b) Write the converse of the proposition.

    c) Which of these is logically equivalent to the proposition?

5. A real valued function f with domain D is said to be **bounded** provided that there exists a real number M such that $|f(x)| \leq M$ for all $x \in D$.

    Negate the above definition by writing what it means for a real valued function f with domain D to be **not bounded**.

6. Suppose our universe of discourse is the set of real numbers. Each of the following quantified statements are true EXCEPT for one. Circle the statement that is false.

- $\forall x\ \exists y \quad x+y=y$
- $\exists x\ \forall y \quad x+y=y$
- $\exists y\ \forall x \quad x \cdot y=x$
- $\forall x\ \exists y \quad x \cdot y=x$

7. Make a truth table to show that $\neg P \rightarrow Q$ is logically equivalent to $P \vee Q$.

8. It is a fact that $P \rightarrow (Q \vee R)$ is logically equivalent to $(P \wedge \neg Q) \rightarrow R$. Show how this logical equivalence can be applied to restate the following proposition:

$$\text{If 3 divides } bc \text{ then 3 divides } b \text{ or 3 divides } c.$$

9. a) List five integers that are congruent to $3 \mod 7$.

   b) $543 \equiv$ _____ $(\text{mod } 11)$ Fill in the blank with an integer between 0 and 11.

   c) If $m \equiv 5 \pmod 8$ and $n \equiv 2 \pmod 8$ then $mn \equiv$ _____ $(\text{mod } 8)$ Fill in the blank with an integer between 0 and 7.

   d) Write the statement "There exists an integer k such that $n = 8k+3$" as an equivalent statement using congruence.

10. Theorem: If 3 does not divide n then 3 divides n^2-1.
If you were planning to prove this theorem using the method of cases based on the Division Algorithm what cases would you use? Only show the cases; don't try to write a formal proof.

11. Suppose a,b,c are integers with $a \neq 0$. Conjecture: If a divides bc then a divides b or a divides c. Show that this conjecture is false by finding a counterexample. (Hint: Try $a=4$ and then find values for b and c that will make the implication false).

12. Let a,b,c be integers with $a \neq 0$ and $b \neq 0$. Prove that if a divides b and b divides c then a divides c. Be sure to point out the integer that you are required to find for the proof.