The Proof Portfolio Project

Description of the Portfolio Project

During the semester, ten problems will be posed to all students. Each student will work on these problems and submit proposed solutions to the professor at the end of the semester. You may bring each of your portfolio problems to the professor one time to be critiqued. The professor will make recommendations about these and give a "temporary grade" for the problem. You then have the opportunity to rewrite and resubmit your problem for the final portfolio.

Important guidelines and rules for the Portfolio Project

- You may not discuss the portfolio problems with anyone except the instructor of the course.
- You may not use any sources to help complete the portfolio problems other than the textbook.
- You may hand in a given portfolio problem to the professor one time to be critiqued.
- No more than two portfolio problems may be submitted for review on a given day.
- The last date to have a portfolio problem critiqued is the beginning of class on Wednesday April 15, 2009.
- The final portfolio is due no later than the beginning of class on Wednesday April 22, 2009.

Grading of the Portfolio Project

The portfolio will be worth a total of 120 points. Each problem will be worth 10 points (for a total of 100 points). In addition, there will be 20 points possible for submission of proofs for review by the professor. To be eligible for the 20 points, a student must do all of the following:

- Submit the first draft of a portfolio problem by Monday February 2, 2009
- Submit the first draft of a second portfolio problem by Monday February 9, 2009
- Submit the first draft of a third portfolio problem by Monday February 16, 2009
- Submit the first draft of a fourth portfolio problem by Monday February 23, 2009
- Submit the first draft of a fifth portfolio problem by Monday March 2, 2009
- Submit the first draft of a sixth portfolio problem by Monday March 16, 2009
- Submit the first draft of a seventh portfolio problem by Monday March 23, 2009
- Submit the first draft of a eighth portfolio problem by Monday March 30, 2009

If you meet these eight deadlines, your score for this part of the portfolio will be 20 out of 20 points. For each deadline that is missed, 2.5 points will be deducted from your score. Please note that these deadlines are independent of each other. For example, if you miss the deadline for submitting the first draft of a second portfolio problem (February 9, 2009), then you still must have three drafts in by February 16, 2009 to meet the third deadline.

Honor System

All work that you submit for the Portfolio Project must be your own work. **This means that you may not discuss the portfolio project with anyone except the instructor of the course and may not use any resources other than the textbook, with the exception that you may look up relevant review material from past mathematics courses.** If you do so, you should cite the book you used and the specific pages you used. You are not, however, permitted to go looking for completed solutions to these problems in any other texts or resources. In particular, using the Internet is completely off limits. Evidence of using Internet sources in your work will result in a minimum penalty of failure for the assignment.

This will also provide me with information regarding how students are doing with each problem. So, if I find that a particular problem is causing more difficulties that anticipated, I can send an email message to all students with hints or points of clarification for that problem.

Following are some (anticipated) questions about this Portfolio. The answers to these questions contain some very important requirements and guidelines for the Portfolio Project.

What other requirements are there for my Portfolio Problems?
The solution for each problem must be written using complete sentences and according to the writing guidelines specified in the text. It must be neat, well organized, and easy to read. Proper grammar, proper sentence and paragraph structure, and correct spelling are necessities.

What happens if I submit an incorrect or incomplete solution?
The professor will return your problem and indicate if it is ready for your Portfolio or if it needs more work. When you submit a solution for a problem before the last day for review, you are asking the professor, "Is this good enough for my Portfolio?" Only the problems turned in by the beginning of class on Wednesday April 22, 2009 will be used to determine your score on the two portions of the proof portfolio.

When can I submit a proposed solution for a problem?
You may submit a problem for review and critique at any time on or before Wednesday April 15, 2009**.**

Should I wait and submit all my problems for review on the last day?
NO!! As soon as you have a proposed solution for a problem, you should write your solution and submit it for review. To encourage this, no more than two portfolio problems may be submitted for review on a given day. **Begin working on your Portfolio Project Immediately!!!**

Can I work with someone else or sources other than the textbook?
The only person you can discuss these problems with is the instructor for the course and the only resource you may use is the textbook. ***Plagiarism is not acceptable*** and will not be tolerated. No credit will be given for the solutions of problems in which plagiarism is involved.

What criteria will be used to judge my proofs?
A proof must be logically and mathematically correct. In addition, it must be written according to the course guidelines as developed in the text and discussed in class.

How will my grade for a given problem be determined?
Each problem in your portfolio will be graded on a 10-point scale with the only possible grades being 10, 9, 6, 3, or 0 points. There will be little partial credit because of the opportunity to submit problems for review, to re-write, and to re-submit. In order to receive full credit for a problem, your solution must be correct, complete, and well written with no spelling or

grammatical errors. The following is a description of the 10-point scale for grading each problem:

Points	Description
10	The proof is correct and written according to the guidelines in the text plus those that follow.
9	The proof is correct but there is a writing mistake.
6	The proof is essentially correct but the solution is not written according to the guidelines.
3	Significant progress has been made in developing and writing a proof for the theorem.
0	Little or no progress has been made in developing a proof for the theorem.

How should I start working on a particular problem?
Before beginning your proof or solution of the problem, you should make a clear statement of exactly what it is that is given in problem (the assumptions) and what is to be proven (the goal). That is, you should analyze the theorem or problem by carefully examining what is given or assumed and precisely what it is that will be proven. In this analysis, you should include any relevant definitions that are needed to clarify the statement of the problem. You should also elaborate on the assumptions made and the strategies that can be used to prove what it is that you are trying to prove. If it is appropriate, you may also include some examples to illustrate the problem.

What are the writing guidelines for writing the solutions of the Portfolio Problems?
To receive full credit, the solution of a portfolio problem must be of collegiate quality and follow the writing guidelines for this course that are given in the textbook. This means that, in addition to demonstrating mastery of the subject matter, the solution should be neat and easy to read, well organized, and use proper grammar and spelling. In addition, a solution must meet the following guidelines:

- You should begin your presentation with a carefully worded statement of the problem. Do not use phrases such as "Show that" or "Prove that". You should state the problem using simple declarative sentences. Following is a typical textbook problem.

  Prove that if n is an integer and n^2 is odd, then n is odd.

  If you were writing a solution to this problem for one of these writing assignments, you should begin in the following manner:

  Theorem: If n is an integer and n^2 is an odd integer, then n is an odd integer.

- All calculations and algebraic manipulations must be clearly shown. By doing so, both you and your professor can follow the process you used to obtain an answer. Without a step-by-step presentation, it may be impossible to understand your solution, or if a mistake is made, it may be impossible to determine where a mistake was made.

- You might start your solution with a short discussion of the strategy that you will use. This is required if you use an indirect method of proof such as a proof by contradiction or use of the contrapositive of a statement. In addition, you should conclude any proof with a statement of what has been proven, or minimally, that the proof is now complete.

Problem One
Suppose that a, b and c are integers. Prove: If c divides $2a-b$ and c divides $b-a$ then c divides a.

Problem Two
Prove that if n is an odd integer then n^2 has the form $8k+1$ where k is an integer.

Problem Three
a) Prove the lemma: For each integer n, 3 divides n if and only if 3 divides n^2.

b) Use the lemma to prove that $\sqrt{3}$ is an irrational number.

Problem Four
A natural number m is called a **perfect square** provided that $m = k^2$ for some integer k. Prove that if n is a natural number and $n \equiv 2 \pmod 3$ then n is not a perfect square.

Problem Five

Recall from probability that the combination formula in terms of factorials is ${}_nC_r = \dfrac{n!}{(n-r)!\,r!}$.

Prove for every natural number n the equation $\sum_{i=0}^{n} {}_nC_i = 2^n$ holds.

Problem Six
Define the sequence $\{f_n\}_{n=1}^{\infty}$ recursively by $f_1 = 1$, $f_2 = 1$, and $f_n = f_{n-1} + f_{n-2}$ for all $n \geq 3$ where n is, of course, a natural number. Evaluate the sum $\sum_{i=1}^{m} (f_i)^2$ for various values of m. Based on your investigation, make a conjecture about a formula for the sum. Prove your conjecture.

Problem Seven
Suppose A, B, C and D are subsets of a universal set U.

Either prove or disprove (by providing a counter example) each of the following:

a) If $A \cap B \neq \varnothing$ and $B \cap C \neq \varnothing$ and $A \cap C \neq \varnothing$ then $A \cap B \cap C \neq \varnothing$.

b) If $A \times B = A \times C$ then $B = C$

c) $(A \times B) \cup (C \times D) \subseteq (A \cup C) \times (B \cup D)$

Problem Eight
Recall that Z is the set of integers and Q is the set of rational numbers. Let
$A = \{(m, n) \mid (m, n) \in Z \times Z,\ n \neq 0\}$ and let $f : A \to Q$ be defined by $f(m, n) = \dfrac{m+n}{n}$.

Is f an injection? Justify your answer.

Is f a surjection? Justify your answer.

Problem Nine
Review the definition of the **image of a set** A **under** f in your textbook**.**

Suppose S and T are sets and $f : S \to T$ is a function. Let $A \subseteq S$.

a) Prove: $f(S) - f(A) \subseteq f(S - A)$.

b) Find a condition on f that would make $f(S) - f(A) = f(S - A)$ then prove that the equation holds using the condition you found.

Problem Ten
Proposition 1: For any real number x there exists a positive integer n such that $n > x$.

Proposition 2: For any real numbers p and q with $q > 0$ there exists a positive integer m such that $mq > p$.

Prove that the preceding two propositions are equivalent.
