Excerpt from syllabus:

Number Theory Portfolio
We will use number theory as the primary subject matter while you are learning proof techniques. Definitions and theorems related to number theory are scattered throughout the text. The number theory portfolio will be where you collect and organize this number theory content. You will be given a list of terms to be defined and a list of key theorems. You will also include in this portfolio specific examples demonstrating the various definitions and theorems.

Further Guidance

- You do not need to type the number theory portfolio. Make it neat. Since it will be accumulated throughout the term, keep it on loose leaf paper in a binder.

- You should organize the number theory portfolio into four units.
  1) Divisibility
  2) Congruence
  3) Prime factorization
  4) Linear Diophantine equations

- The following are a list of definitions, theorems and problems to include in each unit. Your portfolio should include at least these but you may add any other examples or relevant theorems that you chose.

I. Divisibility

1. Suppose a and b are integers and a is not 0. Write the definition of <u>a divides b</u>. Give at least two numerical examples.

2. List the results about divisors given in Theorem 3.1 on page 147 of the text and include a numerical example of each.

3. Do exercise #1 part d) in section 3.1 on page 89.

4. Write a precise statement of the Division Algorithm if the integer b is the divisor and a is the dividend. Apply the Division Algorithm in the following examples:
   1) a = 35 and b = 8
   2) a = 350 and b = 7
   3) a = 48 and b = 12
   4) a = 17 and b = 23

5. Explain how the Division Algorithm allows you to prove theorems about integers by establishing cases.

6. Use the Euclidean Algorithm to find the greatest common divisor for 1416 and 664 and then write the gcd as a linear combination of 1416 and 664.

7. Find a linear combination of 23 and 78 that gives 1. In other words find integers x_o and y_o such that $23 x_o + 78y_o = 1$.

II. Congruence

1. Write the definition of a is congruent to b modulo n and give three numerical examples.

2. List 5 numbers that are greater than 100 and are congruent to 2 modulo 11.

3. Suppose that *a* and *n* are integers and r is the remainder when the Division Algorithm is applied to *a* and *n* (*n* being the divisor). Show that $a \equiv r \pmod{n}$. What are the possible values for *r* in (mod 5) ? in (mod *n*)? These values are called the least positive residue. For example, the least positive residue of 25 in (mod 7) is 4.

4. Find the least positive residue of 234 (mod 10). Try some more examples using (mod 10). What conjecture would you make?

5. List the least positive residues for 2^k in (mod 10) for k = 1,2,…12. What is least common residue of 2^{101} (mod 10)

6. Write out Theorem 3.30 on page 134. Give numerical examples of each part.

7. Write a proof for part 1 of Theorem 3.30 on page 134.

III. Prime numbers

1. Write a precise statement of the Fundamental Theorem of Arithmetic.

2. Factor each of the following into a product of powers of primes.

   A) 360 B) 2006 C) 3200 D) 1331

3. A natural number, n, is square if there exists a natural number m such that $n = m^2$. Write the prime factorization of the following squares.

   A) 16 B) 2025 C) 5481

   Is each number a square? What do you notice about the prime factorizations? Write a general rule that describes how to look at the prime factorization and decide if the number is a square.

4. A natural number is square free if it has no divisors greater than 1 that are squares. For example, 6 is square free since 2, 3, and 6 are not squares but 8 is not square free because 4 divides 8 and 4 is a square. Which of the following are square free?

   A) 75 B) 2006 C) 2584 D) 5^3*7*11^5

5. Describe how you can tell from the prime factorization of a natural number whether or not it is square-free. Give an example.

6. The prime factorization of a natural number n can be written with product notation as $\prod_{k=1}^{r} p_k^{e_k}$. For each of the numbers in problem 2 above tell what r, p_k and e_k are in the product form.

7. Write the prime factorization of 72. Also write the natural number divisors of 72 in a factored form. What you notice about all the divisors?

8. Write the divisors of $7^2 13^3$ in factored form? How many are there?

9. How many natural number divisors does $7^{20}13^{50}$ have?

10. Write a rule for the number of natural number divisors of a natural number n. (Hint: think of n written in the factored form given in problem 6.)

11. Determine the gcd of $3^4 7^5 11*19$ and $3^2 7^7 11^2 *13$

12. Write a rule for how to get the gcd of two integers using the prime factored form of the integers.

13. How can you tell by looking at the prime factored form whether two integers a and b are relatively prime? Show how to apply your rule by giving and an example of a and b that are relatively prime and an example that is not relatively prime.

IV. Linear Diophantine Equations

1. Find all positive integer solutions to the equation 15x + 35y = 2800.

2. Does the equation 255x + 465y = 10000 have any integer solutions? Justify your answer.

3. To solve the congruence $28x \equiv 1 \pmod{31}$ you can write it as a linear Diophantine equation. Show how to do this and solve the equation for $0 \leq x < 31$

4. A student returning from Europe changes his French and Swiss francs into U.S. money. He received 19 cents for each French franc and 59 cents for each Swiss franc and he gets a total of $17.06 in U.S money. How much of each type did he exchange? Is there more than one possibility?