Theorem: For any integers a, b, and c with $a \neq 0$, if a divides b and a divides c, then a divides $b+c$.

Proof: Let a, b, and c be integers with $a \neq 0$. We assume that a divides b and that a divides c. We need to prove that a divides $b+c$. This means that we need to find an integer q such that

$$b+c=aq.$$

Our assumption that a divides both b and c means that there exists integers m and n such that

$$b=am \text{, and} \qquad (1)$$

$$c=an \qquad (2)$$

We can now add equations (1) and (2) to obtain

$$b+c=am+an$$

Using the distributive property on the right-hand side of the above equation yields

$$b+c=a(m+n) \qquad (3)$$

Since m and n are integers, the expression $m+n$ appearing in equation (3) is also an integer by the closure property of addition. By setting

$$q=m+n$$

we have found the required integer mentioned at the beginning of the proof. This proves that a divides $b+c$. Since a, b, and c were arbitrary integers with $a \neq 0$, we have shown that for any integers a, b, and c with $a \neq 0$, if a divides b and a divides c, then a divides $b+c$.

Q.E.D