MAC 2312 Calculus with Analytic Geometry II
TEST 4 – Part Two

Name___________________________________ Score__________

Directions: This is the take-home portion of the test. You must do all the problems contained here <u>individually</u>. You MAY NOT solicit help from anyone. You will get a grade of ZERO for this portion if you are caught cheating. Part one has 36 points and part two has 14 points making a total of 50 points on this test. Your grade will be based on a percentage of the total points.

1. Define a sequence recursively as follows: $a_1 = 1$ and $a_2 = 1$, and for $n \geq 3$, $a_n = a_{n-1} + a_{n-2}$. This is the famous Fibonacci sequence. Now define a new sequence by $b_n = \frac{a_{n+1}}{a_n}$. This new sequence of numbers denoted $\{b_n\}_{n=1}^{\infty}$ converges. Find the <u>exact</u> real number this sequence converges to. (Hint: How do b_n and b_{n-1} relate to each other in the form of an equation? Once you find such an equation, take the limit of both sides and assume that $\lim_{n\to\infty} b_n = \lim_{n\to\infty} b_{n-1} = b$ where b is the real number the sequence converges to. Next solve for b.) (7 points)

2. Find the <u>exact</u> sum of the following telescoping series. Use algebra to justify your work for full credit. (Hint: Perform a partial fraction decomposition and use $\frac{-1}{x+1} = \frac{-1}{2(x+1)} + \frac{-1}{2(x+1)}$.) (7 points)

$$\sum_{n=1}^{\infty} \frac{1}{n^3 + 3n^2 + 2n}$$