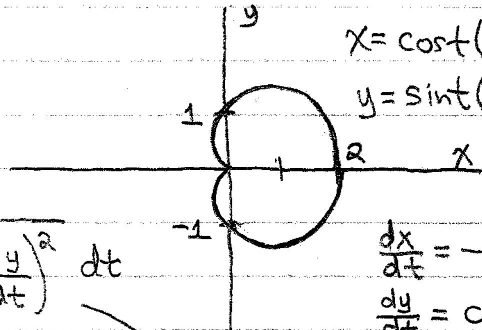
You Gotta Have Heart

①



$$x = \cos t(1+\cos t), \quad y = \sin t(1+\cos t) \quad \Big\rangle \ 0 \le t \le 2\pi.$$

(a cardiod)

double ∡ formulas

$$\frac{dx}{dt} = -\sin t - 2\cos t \sin t = -(\sin t + \sin(2t))$$

$$\frac{dy}{dt} = \cos t + \cos^2 t - \sin^2 t = \cos t + \cos(2t)$$

use $A = \int_\alpha^\beta 2\pi y \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

② $$A = \int_a^b 2\pi y\, ds = 2\pi \int_0^{\pi/2} \sin t(1+\cos t)\sqrt{\left[-(\sin t + \sin(2t))\right]^2 + (\cos t + \cos(2t))^2}\, dt$$

③ $$= 2\pi \int_0^{\pi/2} \sin t(1+\cos t)\sqrt{\sin^2 t + 2\sin t \sin(2t) + \sin^2(2t) + \cos^2 t + 2\cos t\cos(2t) + \cos^2(2t)}\, dt$$

$$= 2\pi \int_0^{\pi/2} \sin t(1+\cos t)\sqrt{2\left[1 + \sin t \sin(2t) + \cos t\cos(2t)\right]}\, dt$$ used $\sin^2\theta + \cos^2\theta = 1$ (twice)

$$= 2\pi \int_0^{\pi/2} \sin t(1+\cos t)\sqrt{2(1 + \underbrace{2\sin^2 t\cos t + \cos^3 t - \sin^2 t \cos t}_{\text{like terms}})}\, dt$$ use double ∡ formulas

$$= 2\pi \int_0^{\pi/2} \sin t(1+\cos t)\sqrt{2(1 + \cos t(\sin^2 t + \cos^2 t))}\, dt$$ combined like terms and factored out cost from remaining last two terms.

$P\sqrt{P} = P^{3/2}$

$$= 2\pi \int_0^{\pi/2} \sin t(1+\cos t)\sqrt{2}\cdot\sqrt{1+\cos t(1)}\, dt$$ used $\sin^2\theta + \cos^2\theta = 1$ and $\sqrt{a\cdot b} = \sqrt{a}\cdot\sqrt{b}$

$$= 2\pi\sqrt{2}\int_0^{\pi/2} \sin t(1+\cos t)^{3/2}\, dt = -2\pi\sqrt{2}\int_2^1 u^{3/2}\, du = 2\pi\sqrt{2}\int_1^2 u^{3/2}\, du = 2\pi\sqrt{2}\left[\frac{2}{5}u^{5/2}\right]_1^2$$

Let $u = 1+\cos t$

$-du = \sin t\, dt$

$$= \frac{4\pi\sqrt{2}}{5}\left[2^{5/2} - 1\right]$$

$$= \boxed{\frac{4\pi}{5}\left(8 - \sqrt{2}\right)}$$ simplified final ans.

a cardiod is heartlike in shape.