Partial Fractions Solutions

1a) $\int \frac{dx}{x+1} = \ln|x+1| + c$

b) $\int \frac{dx}{x+2} = \ln|x+2| + c$

c) $\int \frac{dx}{x^2+4} = \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + c$

d) $\int \frac{x}{x^2+4}\,dx = \frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln(u)$

$= \frac{1}{2}\ln(x^2+4) + c$

$u = x^2+4$

$\frac{1}{2}\,du = x\,dx$

2) $x^4+3x^3+6x^2+12x+8 = (x+1)(x+2)(x^2+4)$

3) $\frac{20x^2}{x^4+3x^3+6x^2+12x+8} = \frac{A}{x+1} + \frac{B}{x+2} + \frac{Cx+D}{x^2+4}$

$20x^2 = A(x+2)(x^2+4) + B(x+1)(x^2+4) + (Cx+D)(x^2+3x+2)$

Set $x=-2$ get $80 = -8B$ so $\boxed{B=-10}$

Set $x=-1$ get $20 = 5A$ so $\boxed{A=4}$

Now equate coefficients of x^3 on both sides and get $0 = 4-10+C$ so $\boxed{C=6}$

Now equate coefficients of x^2 on both sides and get $20 = 8-10+18+D$ so $\boxed{D=4}$

$$\int\left(\frac{4}{x+1} - \frac{10}{x+2} + \frac{6x+4}{x^2+4}\right)dx = \underbrace{4\ln|x+1|}_{\text{Part a}} - \underbrace{10\ln|x+2|}_{\text{Part b}} + 6\underbrace{\int \frac{x}{x^2+4}\,dx}_{\text{Part d}} + 4\underbrace{\int \frac{1}{x^2+4}\,dx}_{\text{Part c}}$$

$$= 4\ln|x+1| - 10\ln|x+2| + 3\ln(x^2+4) + 2\tan^{-1}\left(\frac{x}{2}\right) + C$$

4) $x^4+3x^3+6x^2+12x+8 \overline{\big) x^4+3x^3+26x^2+12x+8}$ with quotient 1

$$\begin{array}{r} x^4+3x^3+26x^2+12x+8 \\ \underline{x^4+3x^3+6x^2+12x+8} \\ 20x^2 = \text{remainder} \end{array}$$

So $\displaystyle\int \frac{x^4+3x^3+26x^2+12x+8}{x^4+3x^3+6x^2+12x+8}\,dx = \int 1 + \frac{20x^2}{x^4+3x^3+6x^2+12x+8}\,dx$

$= x + 4\ln|x+1| - 10\ln|x+2| + 3\ln(x^2+4) + 2\tan^{-1}\left(\frac{x}{2}\right) + C$ (From #3)