Made in the Shade

(1a) $$\frac{\pi(1)^2}{2}+\frac{\pi\left(\frac{1}{2}\right)^2}{2}+\frac{\pi\left(\frac{1}{4}\right)^2}{2}+\dots=\sum_{n=0}^{\infty}\frac{\pi}{2}\left(\frac{1}{2^n}\right)^2=\frac{\pi}{2}\sum_{n=0}^{\infty}\left(\frac{1}{4}\right)^n$$

$$=\frac{\pi}{2}\left[1+\sum_{n=1}^{\infty}\left(\frac{1}{4}\right)^n\right]=\frac{\pi}{2}\left[1+\frac{\frac{1}{4}}{1-\frac{1}{4}}\right]=\frac{\pi}{2}\left[1+\frac{1}{3}\right]=\frac{2\pi}{3}$$

(b) $$\frac{1^2}{4}+\frac{\left(\frac{1}{2}\right)^2}{4}+\frac{\left(\frac{1}{4}\right)^2}{4}+\dots=\frac{1}{4}\sum_{n=0}^{\infty}\left(\frac{1}{2^n}\right)^2=\frac{1}{4}\sum_{n=0}^{\infty}\left(\frac{1}{4}\right)^n=\frac{1}{4}\left[1+\sum_{n=1}^{\infty}\left(\frac{1}{4}\right)^n\right]$$

$$=\frac{1}{4}\left[1+\frac{1}{3}\right]=\frac{1}{3}$$

(2a) $$a_n=\left(\frac{1}{2}\right)^n$$

(2b) $$\sum_{n=1}^{\infty}a_n=\sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^n=\sum_{n=1}^{\infty}\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^{n-1}=\frac{\frac{1}{2}}{1-\frac{1}{2}}=\frac{\frac{1}{2}}{\frac{1}{2}}=1$$