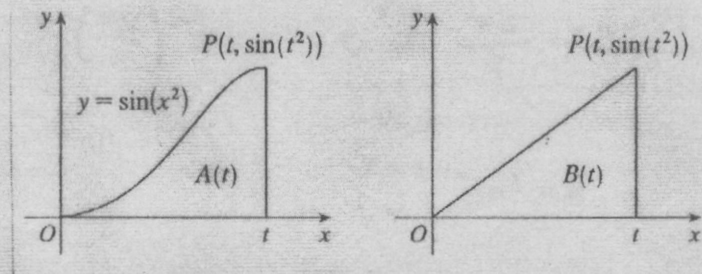
1. The figure below shows two regions in the first quadrant: $A(t)$ is the area under the curve $y = \sin(x^2)$ from 0 to t, and $B(t)$ is the area of the triangle with vertices O, P, and $(t, 0)$. Find $\lim_{t\to 0^+} \frac{A(t)}{B(t)}$. (4 points)



$$A(t) = \int_0^t \sin(x^2)\,dx \qquad B(t) = \frac{1}{2} t \cdot \sin(t^2)$$

$$\lim_{t\to 0^+} A(t) = 0 \qquad \lim_{t\to 0^+} B(t) = 0$$

L'Hospital's Rule

F.T.C.

$$\lim_{t\to 0^+} \left[\frac{\int_0^t \sin(x^2)\,dx}{\frac{1}{2} t \cdot \sin(t^2)} \right] = \lim_{t\to 0^+} \frac{\frac{d}{dt}\left[\int_0^t \sin(x^2)\,dx\right]}{\frac{d}{dt}\left(\frac{1}{2} t \cdot \sin(t^2)\right)} = \lim_{t\to 0^+} \frac{\overbrace{\sin(t^2)}^{\text{tends to 0 as } t\to 0^+}}{\underbrace{\frac{1}{2}\sin(t^2) + t^2\cos(t^2)}_{\text{tends to 0 as } t\to 0^+}}$$

L'Hospital's Rule again

$$= \lim_{t\to 0^+} \frac{2t \cdot \cos(t^2)}{t \cdot \cos(t^2) + 2t \cdot \cos(t^2) - 2t^3 \sin(t^2)}$$

factor/cancel out a t.

direct sub. prop.

$$= \lim_{t\to 0^+} \frac{2\cos(t^2)}{\cos(t^2) + 2\cos(t^2) - 2t^2 \sin(t^2)} = \frac{2\cos(0)}{\cos(0) + 2\cos(0) - 0} = \frac{2}{1+2} = \boxed{\frac{2}{3}}$$