Exponential and Logarithm Worksheet Answers

① $y = \dfrac{e^x}{\sqrt{1-x}}$ quotient rule $y' = \dfrac{\sqrt{1-x}\cdot(e^x)' - e^x(\sqrt{1-x})'}{(\sqrt{1-x})^2}$

chain rule

$$= \frac{\dfrac{e^x\sqrt{1-x}}{1} + \dfrac{e^x}{2\sqrt{1-x}}}{1-x} = \boxed{\frac{2e^x(1-x)+e^x}{2(1-x)^{3/2}}}$$

② $y = x^2 e^{\tan x}$ product rule $y' = 2xe^{\tan x} + x^2(e^{\tan x})'$ chain rule

$$= 2xe^{\tan x} + x^2 \cdot \sec^2 x \cdot e^{\tan x}$$

$$= \boxed{e^{\tan x}\left(2x + (x\cdot\sec x)^2\right)}$$

③ $y = x\cdot\sec(e^x)$ product rule $y' = (1)\cdot\sec(e^x) + x\cdot(\sec(e^x))'$ chain rule

$$= \sec(e^x) + x\cdot\sec(e^x)\cdot\tan(e^x)\cdot(e^x)'$$

$$= \boxed{\sec(e^x)\left[1 + xe^x\tan(e^x)\right]}$$

chain rule

④ $y = \dfrac{1}{e^{2x}+e^{-x}} = (e^{2x}+e^{-x})^{-1}$ power rule $y' = -1(e^{2x}+e^{-x})^{-2}\cdot(e^{2x}+e^{-x})'$

chain rule

$$= -(e^{2x}+e^{-x})^{-2}\cdot(2e^{2x}-e^{-x})$$

$$= \boxed{\frac{e^{-x}-2e^{2x}}{(e^{2x}+e^{-x})^2}}$$

⑤ $y = e^{x^e}$

chain rule

$$y' = e^{x^e}\cdot(x^e)'$$

$$= \underbrace{e\cdot e^{x^e}\cdot x^{e-1}}_{\text{Power rule}} = \boxed{e^{x^e+1}\cdot x^{e-1}}$$

⑥ $y = x^2 \cdot \ln(\cos x)$ product rule $y' = 2x \cdot \ln(\cos x) + x^2 \cdot [\ln(\cos x)]'$ chain rule

$$= 2x\ln(\cos x) + x^2 \cdot \frac{1}{\cos x} \cdot (\cos x)'$$

$$= \boxed{2x\ln(\cos x) - x^2\tan x}$$

⑦ $y = \sqrt{\sin(e^2)} - 9$ derivative of a constant is zero. $\boxed{y' = 0}$

⑧ $y = \ln(x^3 \sin x)$ chain rule/product rule $y' = \frac{1}{x^3 \cdot \sin x} \cdot (x^3 \sin x)'$ chain rule

$$= \frac{3x^2 \sin x + x^3 \cos x}{x^3 \cdot \sin x}$$ Product rule

$$= \frac{3\sin x + x\cos x}{x \sin x}$$ algebra

$$= \boxed{\frac{3}{x} + \cot x}$$ algebra

⑨ $y = e(\ln x)^e$ power/chain rules

chain rule

$$y' = e^2(\ln x)^{e-1} \cdot (\ln x)'$$

$$= \boxed{\frac{e^2(\ln x)^{e-1}}{x}}$$

⑩ $xe^y - ye^x = 1$ product rule twice $(x)'e^y + x(e^y)' - [(y)'e^x + y(e^x)'] = 0$

$$e^y + xe^y \cdot y' - [y'e^x + ye^x] = 0$$

$$e^y + xe^y \cdot y' - y'e^x - ye^x = 0$$

$$xe^y \cdot y' - y'e^x = ye^x - e^y \rightarrow y' = \frac{ye^x - e^y}{xe^y - e^x}$$

plug in $(1,0)$ get $m = \frac{1}{e-1}$

$y - y_1 = m(x - x_1) \leftarrow m = \frac{1}{e-1}$

$x_1 = 1$

$y_1 = 0$

$$\boxed{y = \frac{1}{e-1}(x-1)}$$