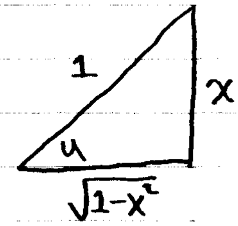
An Indefinite Integral

$$\int e^{\sin^{-1}x}\,dx = \int e^{u}\cos u\,du$$

Substitute:

① Let $x = \sin u \rightarrow u = \sin^{-1}x$

$dx = \cos u\,du$



② Integrate by parts $f = e^{u}$ $g = \sin u$

$f' = e^{u}$ $g' = \cos u$

$$\int e^{u}\cos u\,du = e^{u}\sin u - \int e^{u}\sin u\,du$$

③ Use parts again $f = e^{u}$ $g = -\cos u$

$f' = e^{u}$ $g' = \sin u$

$$\int e^{u}\cos u\,du = e^{u}\sin u - \left[-e^{u}\cos u - \int -e^{u}\cos u\,du\right] = e^{u}\sin u + e^{u}\cos u - \int e^{u}\cos u\,du$$

$$2\int e^{u}\cos u\,du = e^{u}\sin u + e^{u}\cos u \rightarrow \int e^{u}\cos u\,du = e^{u}\left(\frac{\sin u + \cos u}{2}\right)$$

$$= e^{\sin^{-1}x}\left(\frac{\sin(\sin^{-1}x) + \cos(\sin^{-1}x)}{2}\right) = e^{\sin^{-1}x}\left(\frac{x + \sqrt{1-x^{2}}}{2}\right) + C$$