Quiz 7

$$\int \frac{1}{x+4+4\sqrt{x+1}}\,dx \longrightarrow 2\int \frac{1}{u^2-1+4+4u}\,u\,du$$

Let $u=\sqrt{x+1} \Rightarrow u^2-1=x$

$$du=\frac{1}{2\sqrt{x+1}}\,dx$$

$$du=\frac{1}{2u}\,dx$$

$$2u\,du=dx$$

$$= 2\int \frac{u}{u^2+4u+3}\,du$$

$$= 2\int \frac{u}{(u+3)(u+1)}\,du$$

Partial Fractions

$$\frac{u}{(u+3)(u+1)} = \frac{A}{u+3}+\frac{B}{u+1} = \frac{A(u+1)+B(u+3)}{(u+3)(u+1)} = \frac{Au+Bu+A+3B}{(u+3)(u+1)}$$

$$1\cdot u+0 = (A+B)u+(A+3B)$$

$$A+B=1$$
$$A+3B=0$$

$$0A+2B=-1$$

$$B=\frac{-1}{2}$$

$$A=\frac{3}{2}$$

$$= 2\int \frac{3}{2(u+3)}\,du - 2\int \frac{1}{2(u+1)}\,du$$

$$= 3\ln|u+3| - \ln|u+1| + C$$

$$= 3\ln\left|3+\sqrt{x+1}\right| - \ln\left|1+\sqrt{x+1}\right| + C$$