

QUIZ 6

$$\int \frac{x^2 - x + 6}{x^3 + x} dx = \int \frac{x^2 - x + 6}{x(x^2 + 1)} dx = \int \left(\frac{A}{x} + \frac{Bx + C}{x^2 + 1} \right) dx$$

$$\begin{aligned} x^2 - x + 6 &= Ax^2 + A + Bx^2 + Cx \\ &= (A+B)x^2 + Cx + A \end{aligned}$$

Equating coefficients we obtain $A+B=1$ This implies that $B=-5$
 $C=-1$
 $A=6$

$$\text{Hence } \int \frac{x^2 - x + 6}{x^3 + x} dx = \int \left(\frac{6}{x} + \frac{-5x - 1}{x^2 + 1} \right) dx = \int \left(\frac{6}{x} - \frac{5x + 1}{x^2 + 1} \right) dx$$

$$\begin{aligned} &= 6 \int \frac{1}{x} dx - \left[5 \int \frac{x}{x^2 + 1} dx + \int \frac{1}{x^2 + 1} dx \right] \\ &\quad \uparrow \\ &\quad \text{Let } u = x^2 + 1 \\ &\rightarrow = 6 \ln|x| - \frac{5}{2} \ln|x^2 + 1| - \tan^{-1}x + C \end{aligned}$$