

QUIZ 5

Compute: $\int x\sqrt{1-x^4} dx$

$$= \int \sqrt{\sin \theta} \cdot \cos \theta \cdot \frac{\cos \theta}{2\sqrt{\sin \theta}} d\theta$$

$$= \frac{1}{2} \int \cos^2 \theta d\theta$$

$$= \frac{1}{4} \int 1 + \cos(2\theta) d\theta$$

$$= \frac{1}{4} \left[\theta + \frac{1}{2} \sin(2\theta) \right]$$

$$= \frac{1}{4} \left[\theta + \sin \theta \cdot \cos \theta \right]$$

$$= \frac{1}{4} \left[\sin^{-1}(x^2) + \sin(\sin^{-1}(x^2)) \cdot \cos(\sin^{-1}(x^2)) \right]$$

$$= \frac{1}{4} \left[\sin^{-1}(x^2) + x^2 \cdot \sqrt{1-x^4} \right] + C$$

Let $x^2 = \sin \theta$ so $\theta = \sin^{-1}(x^2)$
or

$$x = \sqrt{\sin \theta}$$

$$\text{so } dx = \frac{\cos \theta}{2\sqrt{\sin \theta}} d\theta$$

