QUIZ 3

indet. form $\infty \cdot 0$

$$\lim_{x\to\infty}\left(\frac{x}{x+1}\right)^x = \lim_{x\to\infty} e^{\ln\left(\frac{x}{x+1}\right)^x} \underset{\text{Prop. of logs.}}{=} \lim_{x\to\infty} e^{x\ln\left(\frac{x}{x+1}\right)} \underset{\text{by continuity of } f(x)=e^x}{=} e^{\lim_{x\to\infty} x\cdot\ln\left(\frac{x}{x+1}\right)}$$

indet. form $\frac{0}{0}$

$$= e^{\lim_{x\to\infty}\frac{\ln\left(\frac{x}{x+1}\right)}{1/x}} \underset{\text{Prop of logs.}}{=} e^{\lim_{x\to\infty}\frac{\ln(x)-\ln(x+1)}{1/x}} \underset{\text{L'Horpital's Rule}}{=} e^{\lim_{x\to\infty}\left[\frac{\frac{1}{x}-\frac{1}{x+1}}{-1/x^2}\right]}$$

$$\underset{\text{algebra}}{=} e^{\lim_{x\to\infty}\left(\frac{-x^2}{x^2+x}\right)} \underset{\text{since the degrees of the num. and den. are the same.}}{=} \boxed{e^{-1}}$$

← final ans.