Quiz 15

$$\sum_{n=1}^{\infty} \frac{(4x+1)^n}{n^2}$$

Ratio Test

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \left| \frac{\cancel{(4x+1)^{n+1}}^{(4x+1)}}{(n+1)^2} \cdot \frac{n^2}{\cancel{(4x+1)^n}} \right|$$

$$= \lim_{n\to\infty} \frac{n^2}{(n+1)^2} \cdot |4x+1|$$

$$= |4x+1| \cdot \lim_{n\to\infty} \frac{n^2}{(n+1)^2}$$

$$= |4x+1| \cdot 1$$

$$= |4x+1|$$

If $|4x+1| < 1$ the the series converges. So we solve this inequality.

$$-1 < 4x+1 < 1$$

The radius of convergence is $R = \frac{1}{4}$.

$$-2 < 4x < 0$$

$$-\frac{1}{2} < x < 0$$

we check the endpoints. with $x=0$ the series becomes $\sum_{n=1}^{\infty} \frac{1}{n^2}$ which converges because its a p-series with $p=2>1$. with $x=-\frac{1}{2}$ the series becomes $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ which also converges by the alternating series test. i.e. $f(x) = \frac{1}{x^2}$ is a decreasing function on the interval $(1,\infty)$ and $\lim_{n\to\infty} \frac{1}{n^2} = 0$. The interval of convergence is $\left[-\frac{1}{2}, 0\right]$.