

Quiz 10 (Pg. 657)
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a) The solution to $\frac{dT}{dt} = k(T - T_s)$ subject to $T(0) = T_0$

is $T = (T_0 - T_s)e^{kt} + T_s$. Here $T_0 = 185$ and $T_s = 75$

so that $T = 110e^{kt} + 75$. We know that $T(\frac{1}{2}) = 150$

so $150 = 110e^{\frac{k}{2}} + 75$. This implies that $k = 2 \ln\left(\frac{15}{22}\right)$

so $T = 110e^{2 \ln\left(\frac{15}{22}\right)t} + 75$. This means $T\left(\frac{3}{4}\right) \approx 136.93^\circ \text{F}$

b) solve $100 = 110e^{2 \ln\left(\frac{15}{22}\right)t} + 75$. Here $t = 1.934248712$ hours

so $t \approx 116$ minutes