MAC 2311 Calculus with Analytic Geometry I
Test 4

Name KEY Score 88

★ must use principles of calculus ★

1. A car is traveling at 90 km/h when the driver sees a road block 125m ahead and slams on the brakes. What constant deceleration is required in order to stop the car at the time the car reaches the road block? (Hint: Be sure to convert everything to the same units. In this case, it is easiest to use meters and seconds). (8 points)

$90\,km/h = \frac{90000}{3600}\,m/s = \frac{150}{6} = \frac{50}{2} = 25\,m/s$

$a(t) = k$

$V(t) = kt + c_1 \quad v(0) = 25 \Rightarrow c_1 = 25$

$v(t) = kt + 25$

$s(t) = \frac{k}{2}t^2 + 25t + c_2 \quad s(0) = 0 \Rightarrow c_2 = 0$

$s(t) = \frac{k}{2}t^2 + 25t \Rightarrow s(t) = -\frac{5}{4}t^2 + 25t$

set $v(t) = 0$

$kt + 25 = 0$

$t = \frac{-25}{k}$

Now

$s\left(\frac{-25}{k}\right) = 125$

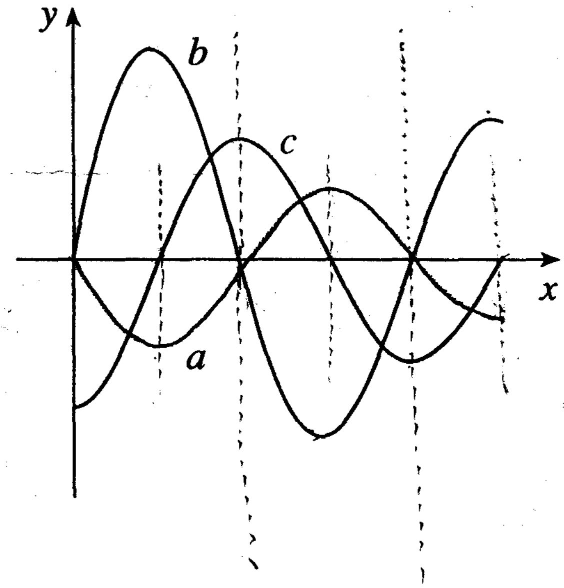
$\frac{k}{2}\left(\frac{-25}{k}\right)^2 + 25\left(\frac{-25}{k}\right) = 125$

$\frac{625}{2k} - \frac{625}{k} = 125$

$\frac{-625}{2k} = 125 \Rightarrow -625 = 250k \Rightarrow k = \frac{-625}{250}$

$= -2.5\,m/s^2$

2. The figure below shows the graphs of f, f', *and* $\int_0^x f(t)dt$. Identify each graph, and explain your choices. Be sure to include in your explanation appropriate terminology such as intervals of increase/decrease, to the left or right of..., critical numbers etc. You may label important aspects of the graphs in the diagram and use your labels as points of reference in your explanation. (6 points)



$a = \int_0^x f(t)\,dt$

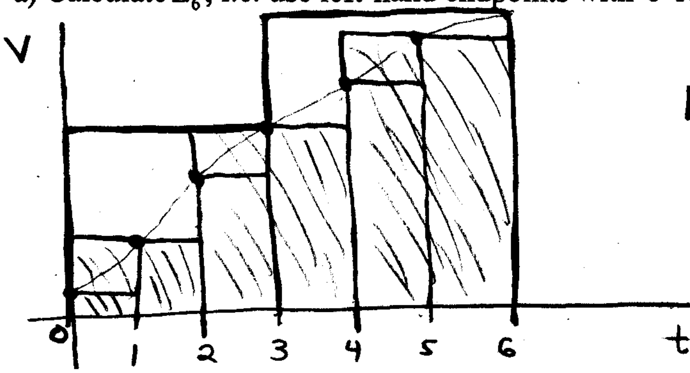
$c = f(x)$

$b = f'(x)$

3. The velocity of a vehicle is measured every second and recorded in the table below. Estimate the distance traveled by the vehicle during the six second interval in three ways as outlined below. You may wish to sketch a rough graph of the data points.

time (sec)	0	1	2	3	4	5	6
velocity (ft./sec)	2	10	24	36	46	54	60

a) Calculate L_6, i.e. use left-hand endpoints with 6 rectangles. (3 points)



$L_6 = 2+10+24+36+46+54$

$= 172$

★ Right ★

b) Calculate R_2, i.e. use ~~left~~-hand endpoints with 2 rectangles. (2 points)

$R_2 = 3 \cdot 36 + 3 \cdot 60$

$= 108 + 180$

$= 288$

c) Calculate M_3, i.e. use midpoints with 3 rectangles. (3 points)

$M_3 = 2(10+36+54)$

$= 200$

4. A particle moves with an acceleration given by the function $a(t) = 10 + 3t - 3t^2$. Two initial conditions, which are $s(0) = 0$ and $s(2) = 10$, are also given. Using this information find the position function of the particle. This means to find an explicit formula for $s(t)$. (8 points)

$$a(t) = -3t^2 + 3t + 10$$

$$v(t) = \frac{-3t^3}{3} + \frac{3}{2}t^2 + 10t + c$$

$$v(t) = -t^3 + \frac{3}{2}t^2 + 10t + c$$

$$s(t) = \frac{-1}{4}t^4 + \frac{1}{2}t^3 + 5t^2 + ct + d$$

$$s(0) = d = 0$$

$$s(t) = \frac{-1}{4}t^4 + \frac{1}{2}t^3 + 5t^2 + ct$$

$$s(2) = \frac{-1}{4}(2)^4 + \frac{1}{2}(2)^3 + 5(2)^2 + 2c = 10$$

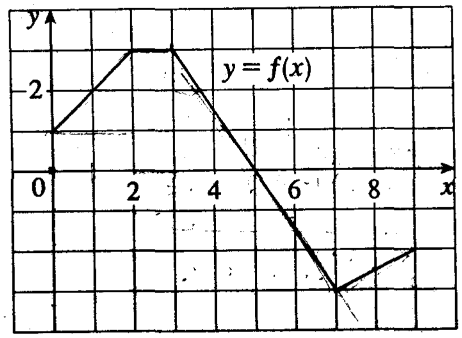
$$= -4 + 4 + 20 + 2c = 10$$

$$= 20 + 2c = 10 \Rightarrow 2c = -10 \Rightarrow c = -5$$

$$\boxed{s(t) = \frac{-1}{4}t^4 + \frac{1}{2}t^3 + 5t^2 - 5t}$$

5. Let $g(x) = \int_0^x f(t)dt$, where f is the function whose graph is show below.

$-\frac{6}{4} = -\frac{3}{2}$



a) Evaluate $g(1)$, $g(3)$, $g(5)$, $g(7)$ and $g(9)$. (3 points)

$g(1) = 1.5$, $g(3) = 7$, $g(5) = 10$, $g(7) = 7$, $g(9) = 2$

b) Find the intervals of increase and decrease for g. (2 points)

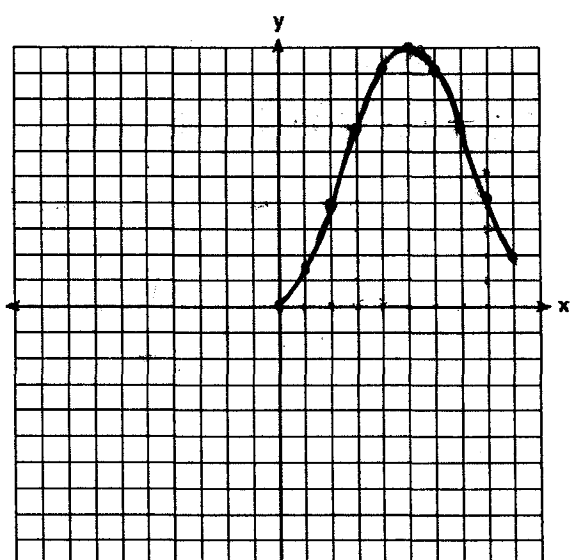
inc. on $(0, 5)$

dec. on $(5, 9)$

c) What type of local extreme value (min. or max.) does g have and at what x value does it occur? (2 points)

g has a local max. at $x=5$

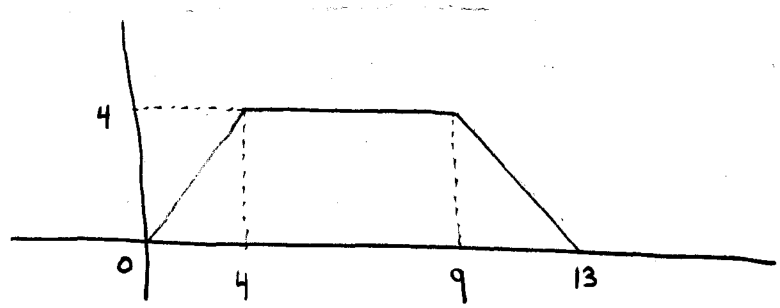
d) Sketch a rough graph of $y = g(x)$ on the graph below. (3 points)



6. Calculate $\int_0^{13} g(x)dx$, where g is the piecewise defined function given by

$$g(x) = \begin{cases} x & for \quad 0 \le x < 4 \\ 4 & for \quad 4 \le x < 9 \\ 13 - x & for \quad 9 \le x \le 13 \end{cases}$$

You may find it helpful to sketch a graph of this function first before you calculate the definite integral. (8 points)



$$\int_0^{13} g(x)\,dx = 8 + 20 + 8 = 36$$

7. Use the definition of the definite integral given by $\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$ to evaluate $\int_1^4 (9-2x)\,dx$. You may use the fact that $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$. (10 points)

$$\lim_{n\to\infty} \sum_{i=1}^{n} (9-2x_i)\Delta x$$

$$\Delta x = \frac{4-1}{n} = \frac{3}{n}$$

$$x_i = a + i\Delta x = 1 + \frac{3i}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{21}{n} - \frac{18i}{n^2}$$

$$9-2x_i = 9-2\left[1+\frac{3i}{n}\right]$$

$$= 9-2-\frac{6i}{n} = 7-\frac{6i}{n}$$

give them

$$= \lim_{n\to\infty}\left[\frac{21}{n}\sum_{i=1}^{n} 1 - \frac{18}{n^2}\sum_{i=1}^{n} i\right]$$

$$= \lim_{n\to\infty}\left[\frac{21}{n}\cdot\frac{n}{1} - \frac{18}{n^2}\cdot\frac{n(n+1)}{2}\right]$$

$$= \lim_{n\to\infty}\left[21 - \frac{9n(n+1)}{n^2}\right]$$

$$= \lim_{n\to\infty}\left[21 - \frac{9(n^2+n)}{n^2}\right]$$

$$= \lim_{n\to\infty}\left[21 - 9\left(1+\frac{1}{n}\right)\right]$$

$$= 21-9$$

$$= 12$$

Check $\int_1^4 9-2x\,dx = 9x-x^2\Big]_1^4$

$$= (36-16)-(9-1)$$

$$= 20-8$$

$$= 12 \checkmark$$

8. The velocity function (in meters per second) for a particle moving along a straight line is given by $v(t) = t^2 - 2t - 8$ for $1 \le t \le 6$.

a) Find the displacement of the particle during the indicated time period. (3 points)

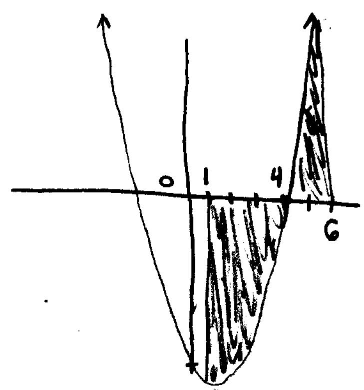
$$\int_1^6 v(t)\,dt = \int_1^6 (t^2-2t-8)\,dt = \frac{1}{3}t^3 - t^2 - 8t \Big]_1^6 = -12 - \frac{-26}{3} = \frac{-10}{3}$$

b) Find the total distance traveled by the particle during the given time interval. (5 points)

$$t^2-2t-8=0$$

$$(t-4)(t+2)=0$$

$$t=4,-2$$



$$\text{distance traveled} = \left|\int_1^4 v(t)\,dt\right| + \left|\int_4^6 v(t)\,dt\right| = |-18| + \left|\frac{44}{3}\right| = 18 + \frac{44}{3} = \frac{98}{3}$$

$$\frac{1}{3}t^3 - t^2 - 8t\Big]_1^4 = \frac{-80}{3} - \frac{-26}{3} = -18$$

$$\frac{1}{3}t^3 - t^2 - 8t\Big]_4^6 = -12 - \frac{-80}{3} = \frac{44}{3}$$

★careful of signs!

9. Find the area of the region that lies under the graph of $y = 2\sin(x) - \sin(2x)$ assuming that $0 \le x \le \pi$. (5 points)

$$\int_0^{\pi} (2\sin x - \sin 2x)\,dx = 2\int_0^{\pi} \sin x\,dx - \int_0^{\pi} \sin 2x\,dx$$

$$= 2\Big[-\cos x\Big]_0^{\pi} - \Big[\frac{-1}{2}\cos 2x\Big]_0^{\pi}$$

$$= 2\Big[1 - {}^{-}1\Big] - \Big[\frac{-1}{2} - \frac{-1}{2}\Big]$$

$$= 2(2) - (0) = 4$$

10. Use the substitution method to evaluate the definite integral $\int_0^1 x^2(1+2x^3)^5\,dx$.

(5 points)

① $u = 2x^3 + 1$

② $\frac{du}{dx} = 6x^2 \Rightarrow \frac{du}{6} = x^2 dx$

$$\frac{1}{6}\int_1^3 u^5\,du = \frac{1}{6}\Big[\frac{1}{6}u^6\Big]_1^3 = \frac{1}{6}\Big[121.5 - \frac{1}{6}\Big] = \frac{182}{9}$$

11. Calculate each of the following using the Fundamental Theorem of Calculus in conjunction with the properties of integrals and rules for integration. (12 points)

a) $\int_0^1 (3x^2 - 2x + 1)\,dx = x^3 - x^2 + x \Big]_0^1 = 1 - 0 = 1$

b) $\frac{d}{dx}\int_{-3}^{x} \frac{\sin u}{u}\,du = \frac{\sin x}{x}$

c) $\int_{3\pi/2}^{2\pi} \cos\theta\,d\theta = \sin(2\pi) - \sin\left(\frac{3\pi}{2}\right) = 0 - {}^{-}1 = 1$

d) $\int\left(x^{4/5} + \frac{3}{x^2}\right)dx = \frac{x^{4/5+1}}{4/5+1} + \frac{3x^{-2+1}}{-2+1} = \frac{5}{9}x^{9/5} - \frac{3}{x} + C$

e) $\int (3x^2 - 1)(2x - 5)\,dx = \int (6x^3 - 15x^2 - 2x + 5)\,dx$

$= \frac{3}{2}x^4 - 5x^3 - x^2 + 5x + C$