MAC 2311 CALCULUS WITH ANALYTIC GEOMETRY ONE
TEST 2

Name KEY Score 72

Directions: Answer each question, showing ALL your work for full credit.

1. The derivative of $f(x) = \dfrac{x}{\sqrt{x^2+4}}$ can be written in the form $f'(x) = \dfrac{k}{(x^2+4)^m}$ where k and m are real numbers. Find the exact values for k and m. (3 points)

$$f'(x) = \frac{\dfrac{\sqrt{x^2+4}}{1} - \dfrac{\cancel{2}x^2}{\cancel{2}\sqrt{x^2+4}}}{\left(\sqrt{x^2+4}\right)^2} = \frac{\dfrac{x^2+4-x^2}{\sqrt{x^2+4}}}{x^2+4} = \frac{4}{(x^2+4)^{3/2}}$$

quotient Rule

algebra

algebra

k

m

So $\boxed{k=4 \text{ and } m=\frac{3}{2}}$

Calculate the derivative of $y = \pi^2 x - 5x^{\sqrt{2}} + \dfrac{3}{x^{-1}} + \sqrt{2} - \pi^2$. (3 points)

$$\boxed{y' = \pi^2 - 5\sqrt{2}\,x^{\sqrt{2}-1} + 3}$$

Show, step by step, that the derivative of $y = 2\sqrt{x}\sin(\sqrt{x}) + 2\cos(\sqrt{x})$ is $y' = \cos(\sqrt{x})$. (3 points)

Product Rule

$$y' = 2 \cdot \frac{1}{2\sqrt{x}} \sin(\sqrt{x}) + 2\sqrt{x} \cdot \cos(\sqrt{x}) \cdot \frac{1}{2\sqrt{x}} - 2\sin(\sqrt{x}) \cdot \frac{1}{2\sqrt{x}}$$

$$y' = \frac{1}{\sqrt{x}} \sin(\sqrt{x}) + \frac{2\sqrt{x}\cos(\sqrt{x})}{2\sqrt{x}} - \frac{1}{\sqrt{x}} \sin(\sqrt{x})$$

$$y' = \frac{2\sqrt{x}\cos(\sqrt{x})}{2\sqrt{x}}$$

$$y' = \cos(\sqrt{x})$$

The derivative of $g(x) = \sec x \tan x$ can be written in the form $g'(x) = m\sec^k x - \sec x$ where k and m are real numbers. Find the exact values for k and m. Hint: use the trigonometric identity $1 + \tan^2 x = \sec^2 x$. (3 points)

$$g'(x) = (\sec x \cdot \tan x)(\tan x) + (\sec x)(\sec^2 x)$$ by product rule

$$= \sec x \cdot \tan^2 x + \sec^3 x$$

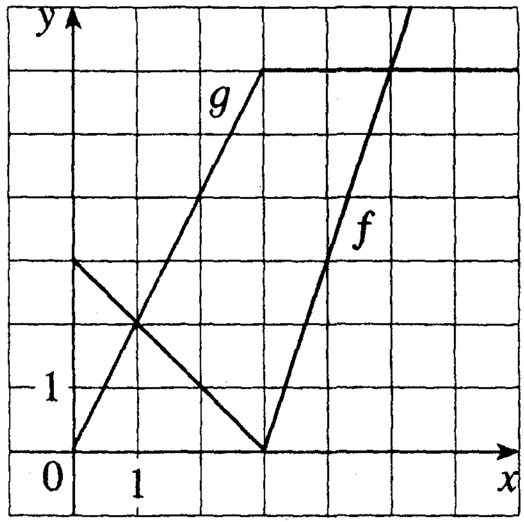
$$= \sec x(\sec^2 x - 1) + \sec^3 x$$

$$= \sec^3 x - \sec x + \sec^3 x$$

$$= \underset{\uparrow m}{2}\sec^{\overset{k}{3}} x - \sec x$$

So $\boxed{m=2 \text{ and } k=3}$

2. Let f and g be functions whose graphs are shown below. Suppose that $P(x) = f(x) \cdot g(x)$ and $C(x) = f(g(x))$. Calculate, step by step, the values of $P'(2)$ and $C'(2)$. (6 points)



$P'(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$

$P'(2) = f'(2) \cdot g(2) + f(2) \cdot g'(2) = (-1)(4) + (1)(2) = -4 + 2 = \boxed{-2}$

AND

$C'(x) = f'[g(x)] \cdot g'(x)$

$C'(x) = f'[g(2)] \cdot g'(2) = f'[4] \cdot g'(2) = 3 \cdot 2 = \boxed{6}$

3. For what values of x on the interval $[0, 4\pi]$ does the graph of $f(x) = x + 2\sin x$ have a horizontal tangent? Note: you are going around the unit circle twice. (6 points)

$$f'(x) = 1 + 2\cos x = 0 \quad \Rightarrow \quad \cos x = \frac{-1}{2}$$

S	A
T	C

cos x is negative in quadrants II and III.

The reference angle is $\theta = \frac{\pi}{3}$.

answers are: $\pi - \frac{\pi}{3}$, $\pi + \frac{\pi}{3}$, $3\pi - \frac{\pi}{3}$, and $3\pi + \frac{\pi}{3}$

OR simply

$\frac{2\pi}{3}$, $\frac{4\pi}{3}$, $\frac{8\pi}{3}$, and $\frac{10\pi}{3}$

4. Find the equation of the tangent line to the graph of $y = \sin(\sin x)$ at $x = \pi$. (6 points)

$$\frac{dy}{dx} = \cos(\sin x) \cdot \cos x$$

$$\left.\frac{dy}{dx}\right|_{x=\pi} = \cos(\sin \pi) \cdot \cos \pi = \cos(0) \cdot \cos \pi = 1 \cdot (-1) = -1$$ ← slope number for tangent line

Plug $x=\pi$ into the original function to obtain $y = \sin(\sin \pi) = \sin(0) = 0$

so $(\pi, 0)$ is a point on the graph.

Use $y - y_1 = m(x - x_1)$

$$y - 0 = -1(x - \pi)$$

$$\boxed{y = -x + \pi}$$

5. A particle moves along a straight line according to the equation $s = t^3 - 9t^2 + 15t + 10$ where $t \geq 0$, t is measured in seconds and s in feet.

a) What is the velocity of the particle after 3 seconds. (1 points)

$v(t) = 3t^2 - 18t + 15$

$v(3) = \boxed{-12 \text{ ft./sec.}}$

b) When is the particle at rest? (2 points)

Set $v(t) = 3t^2 - 18t + 15 = 0$ so $\boxed{t = 5 \text{ and } t = 1}$

$= 3(t^2 - 6t + 5) = 0$

$= 3(t-5)(t-1) = 0$

c) When is the particle moving in the positive direction? (1 points)

Set $v(t) > 0$

This happens when $\boxed{0 < t < 1 \text{ and } t > 5}$

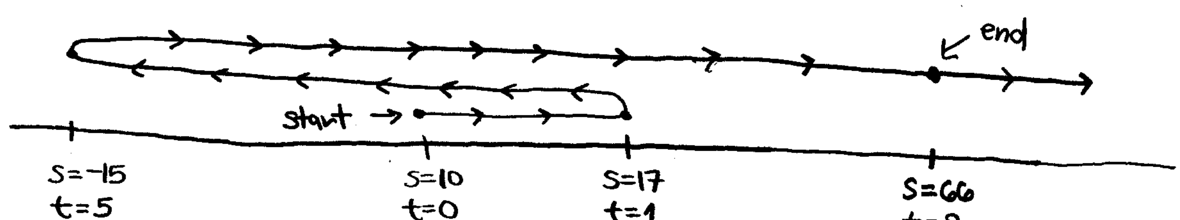
d) When is the particle slowing down? (3 points)

$a(t) = 6t - 18 = 6(t-3)$

This happens when $a(t) > 0$ and $v(t) < 0$ OR $a(t) < 0$ and $v(t) > 0$ so $\boxed{0 < t < 1 \text{ and } 3 < t < 5}$

Note: $a(t) > 0$ when $t > 3$

e) Draw a diagram illustrating the motion of the particle. (3 points)



f) What is the total distance traveled by the particle during the first 8 seconds? (2 points)

$7 + 32 + 81 = \boxed{120 \text{ feet}}$

6. Find $\frac{dy}{dx}$ by implicit differentiation given the equation $1-\cos(x\,y^2)=x$. Express your final answer as a SINGLE fraction. (8 points)

$$\sin(xy^2)\cdot\left[y^2+2xyy'\right]=1$$

chain rule

Product rule

$$y'=\frac{\frac{1}{\sin(xy^2)}-\frac{y^2}{1}}{2xy}$$

$$\boxed{\frac{dy}{dx}=\frac{1-y^2\sin(xy^2)}{2xy\sin(xy^2)}}$$

7. Determine the coefficients a, b, and c so that each of the following is satisfied. (6 points)

- $p(x) = ax^2 + bx + c$
- $p(2) = 5$
- $p'(2) = 3$
- $p''(2) = 2$

$p'(x) = 2ax + b$

$p''(x) = 2a$

So $p''(2) = 2a = 2 \Rightarrow \boxed{a = 1}$

So $p'(x) = 2x + b$

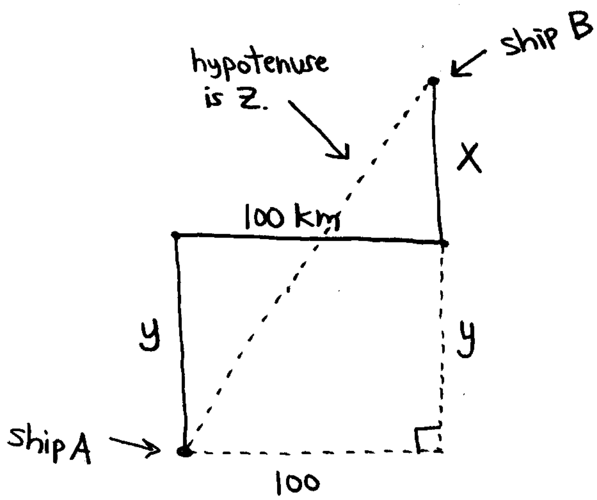
and $p'(2) = 4 + b = 3 \Rightarrow \boxed{b = -1}$

So $p(x) = x^2 - x + c$

and $p(2) = 4 - 2 + c = 5 \Rightarrow \boxed{c = 3}$

So $p(x) = x^2 - x + 3$

8. At noon, ship A is 100 km west of ship B. Ship A is sailing south at 35 km/hour and ship B is sailing north at 25 km/h. How fast is the distance between the ships changing at 4:00 p.m.? Draw a picture a part of your answer. (10 points)



$$100^2+(x+y)^2=z^2$$

we know:

$x=25(4)=100$ km

$y=35(4)=140$ km

$z=260$

$\frac{dx}{dt}=35$ km/hr

$\frac{dy}{dt}=25$ km/hr

differentiate with respect to time $\quad 0+2(x+y)\cdot\left(\frac{dx}{dt}+\frac{dy}{dt}\right)=2z\cdot\frac{dz}{dt}$

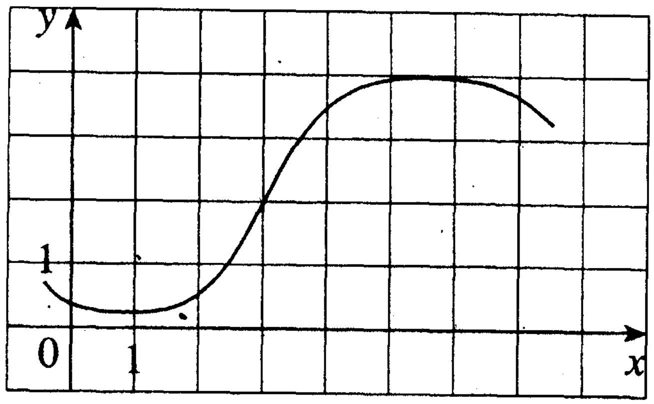
Plug in the known values $\quad 2(100+140)(35+25)=2(260)\cdot\frac{dz}{dt}$

more simply: $14400=260\cdot\frac{dz}{dt}$

so $\frac{dz}{dt}=\frac{14400}{260}\approx 55$ km/hr

9. For the function f whose graph is shown below, arrange the following numbers in increasing order. (6 points)

$0 \quad 1 \quad f'(2) \quad f'(3) \quad f'(5) \quad f'(7)$ ← negative number



$f'(7)$	0	$f'(5)$	$f'(2)$	1	$f'(3)$
smallest					greatest

--------------------------End of Test-----------------------------