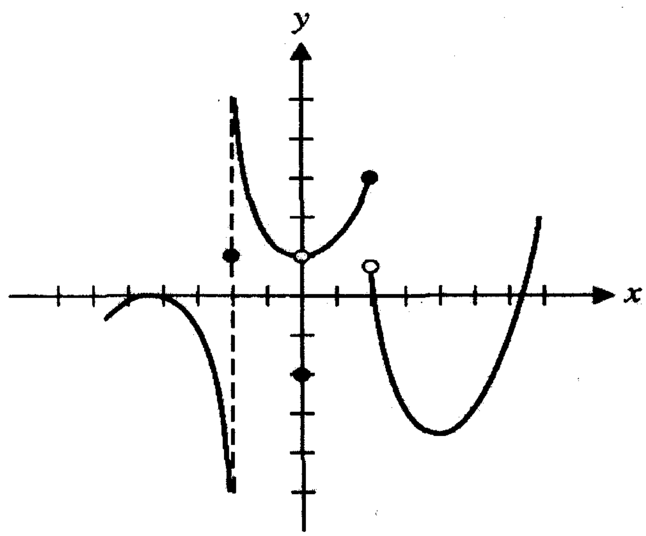
MAC 2311 CALCULUS WITH ANALYTIC GEOMETRY I
TEST 1

Name KEY Score 77

Directions: Answer each question showing ALL work for full credit.

1. Use the graph of the function $y = f(x)$ below to find each limit or functional value. If the limit or functional value does not exist or is undefined, so state. (6 points)



$\lim_{x\to -2^-} f(x) = -\infty$ $\lim_{x\to 2^-} f(x) = 3$ $\lim_{x\to 2^+} f(x) = 1$ $\lim_{x\to 2} f(x) =$ D.N.E

$\lim_{x\to 0^+} f(x) = 1$ $\lim_{x\to 0} f(x) = 1$ $f(-2) = 1$ $f(0) = -2$

$\lim_{x\to a} f(x)$ exists for ALL real numbers a EXCEPT for -2 and 2. (1 point)

$f(x)$ is continuous at ALL real numbers EXCEPT for $-2, 2,$ and 0. (1 point)

2. Sketch the graph of a function that satisfies all of the conditions below. (8 points)

- $\lim\limits_{x \to 3^+} f(x) = 4$

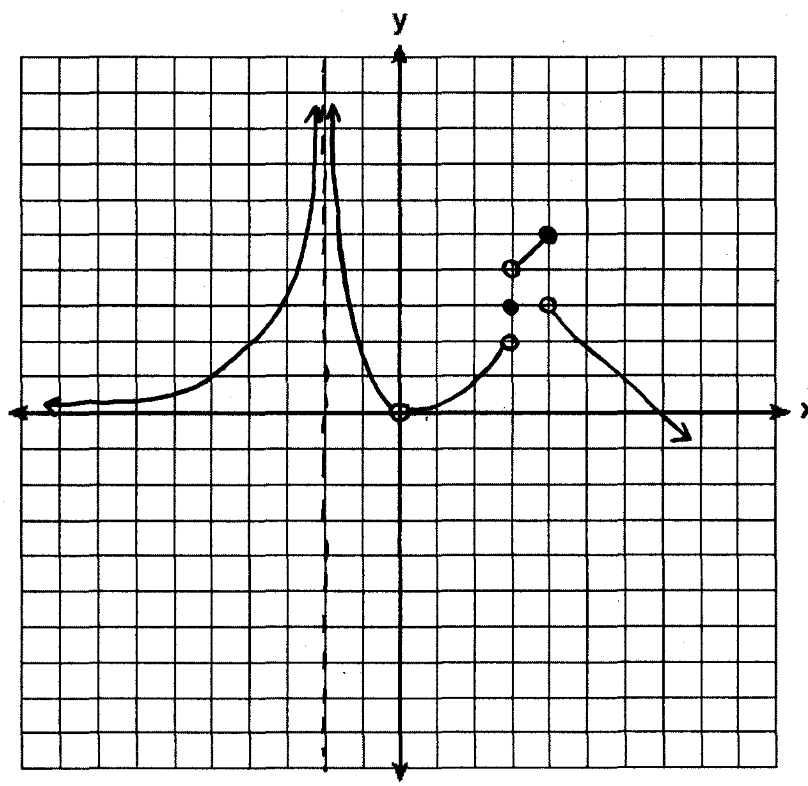
- $\lim\limits_{x \to 3^-} f(x) = 2$

- $\lim\limits_{x \to -2} f(x) = \infty$

- $f(3) = 3$

- $f(0)$ is undefined

- $f(x) = -x + 7$ for $x > 4$



This is $\underline{\underline{\text{ONE}}}$ Possible graph.

3. Compute both of the following limits, if they exist. (8 points)

$$\lim_{x\to -1}\left(\frac{x^2+3x+2}{x^2-x-2}\right)$$

$$= \lim_{x\to -1} \frac{(x+2)(x+1)}{(x-2)(x+1)}$$

$$= \lim_{x\to -1} \frac{(x+2)}{(x-2)}$$

$$\overset{\text{D.S.P.}}{=} \frac{-1+2}{-1-2} = \boxed{\frac{-1}{3}}$$

$$\lim_{x\to 0}\left(\frac{\sqrt{1+x}-1}{x}\right)$$

$$= \lim_{x\to 0} \frac{\sqrt{1+x}-1}{x} \cdot \frac{\sqrt{1+x}+1}{\sqrt{1+x}+1}$$

$$= \lim_{x\to 0} \frac{(1+x)-1}{x\left[\sqrt{1+x}+1\right]}$$

$$= \lim_{x\to 0} \frac{1}{\sqrt{1+x}+1} \overset{\text{D.S.P}}{=} \frac{1}{\sqrt{1+0}+1} = \boxed{\frac{1}{2}}$$

4. A cardiac monitor is used to compile the number of heartbeats for a patient. The values in the table below show the cumulative number of heartbeats H after t ~~seconds~~ minutes. Here H is a function of time. For example $H(38) = 2661$.

t (min.)	36	38	40	42	44	46
H	2530	2661	2806	2948	3080	3202

Part a) If P is the point (42, 2948) on the graph of H, find the slope of the secant line PQ where Q is the point on the graph when $t = 46$. (2 points)

$$m_{sec} = \frac{3202 - 2948}{46 - 42} = \frac{254}{4} = \boxed{63.5}$$

Part b) Give the <u>best</u> possible estimate for the slope of the tangent line at P by averaging the slopes of two secant lines. (2 points)

$$\frac{\frac{3080-2948}{44-42} + \frac{2948-2806}{42-40}}{2} = \boxed{68.5}$$

Part c) Write a sentence or two describing what the equation $H'(42) = 68$ means in layman's terms. Be specific and including units of measurement. (2 points)

42 minutes after being hooked up to a cardiac monitor, the patient has a heart rate of 68 beats per minute.

Part d) According to the table, during which time interval did the patient have the highest heart rate? (2 point)

$$38 \le t \le 40$$

5. The definition for the derivative of a function f is given by

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

a. Use this definition to find the derivative of the function $f(x) = -\frac{1}{x}$ (8 points)

$$f'(x) = \lim_{h \to 0} \left[\frac{\frac{-1}{x+h} + \frac{1}{x}}{h} \right] = \lim_{h \to 0} \left[\frac{-x + x + h}{x(x+h) \cdot h} \right]$$

$$= \lim_{h \to 0} \left[\frac{1}{x(x+h)} \right]$$

$$\overset{\text{D.S.P.}}{=} \boxed{\frac{1}{x^2}}$$

b. Using your result from the first part, find the equation of the tangent line to the graph of f at the point $(0.5, -2.0)$ (2 points)

$f'(0.5) = 4$ Use $y - y_1 = m(x - x_1)$

$$y + 2 = 4(x - 0.5)$$

$$y = 4x - 2 - 2$$

$$\boxed{y = 4x - 4}$$

6. Find the values for a and b that will make g continuous everywhere. (~~8~~ 6 points)

$$g(x) = \begin{cases} 3x+1 & \text{if } x < 2 \\ ax+b & \text{if } 2 \le x < 5 \\ x^2 & \text{if } x \ge 5 \end{cases}$$

we need $g(x)$ cont. at $x=2$ and $x=5$ since it is cont. everywhere else.

This means we need $\lim\limits_{x \to 2} g(x) = g(2) = 2a+b$. Now $\lim\limits_{x \to 2^-} g(x) = \lim\limits_{x \to 2^-} (3x+1) \overset{\text{D.S.P}}{=} 7$

we set $2a+b=7$.

Similarly, we need $\lim\limits_{x \to 5} g(x) = g(5) = 25$. Now $\lim\limits_{x \to 5^-} g(x) = \lim\limits_{x \to 5^-} (ax+b) \overset{\text{DSP}}{=} 5a+b$

we set $5a+b=25$. Next, we solve the system of linear equations $\begin{cases} 2a+b=7 \\ 5a+b=25 \end{cases}$

to obtain $a=6$ and $b=-5$. See below

$$\begin{array}{r} 5a+b=25 \\ -\quad 2a+b=7 \\ \hline 3a = 18 \end{array}$$

$$\frac{3a}{3} = \frac{18}{3}$$

$$a=6$$

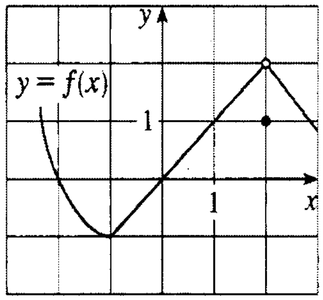
Substit. $2(6)+b=7$

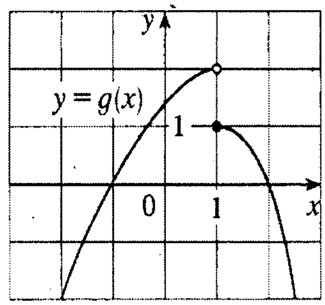
$12+b=7$

$b=7-12$

$b=-5$

7. The graphs of f and g are given below. Use them to evaluate each limit, if it exists. If the limit does not exist write DNE in the space provided. (9 points)



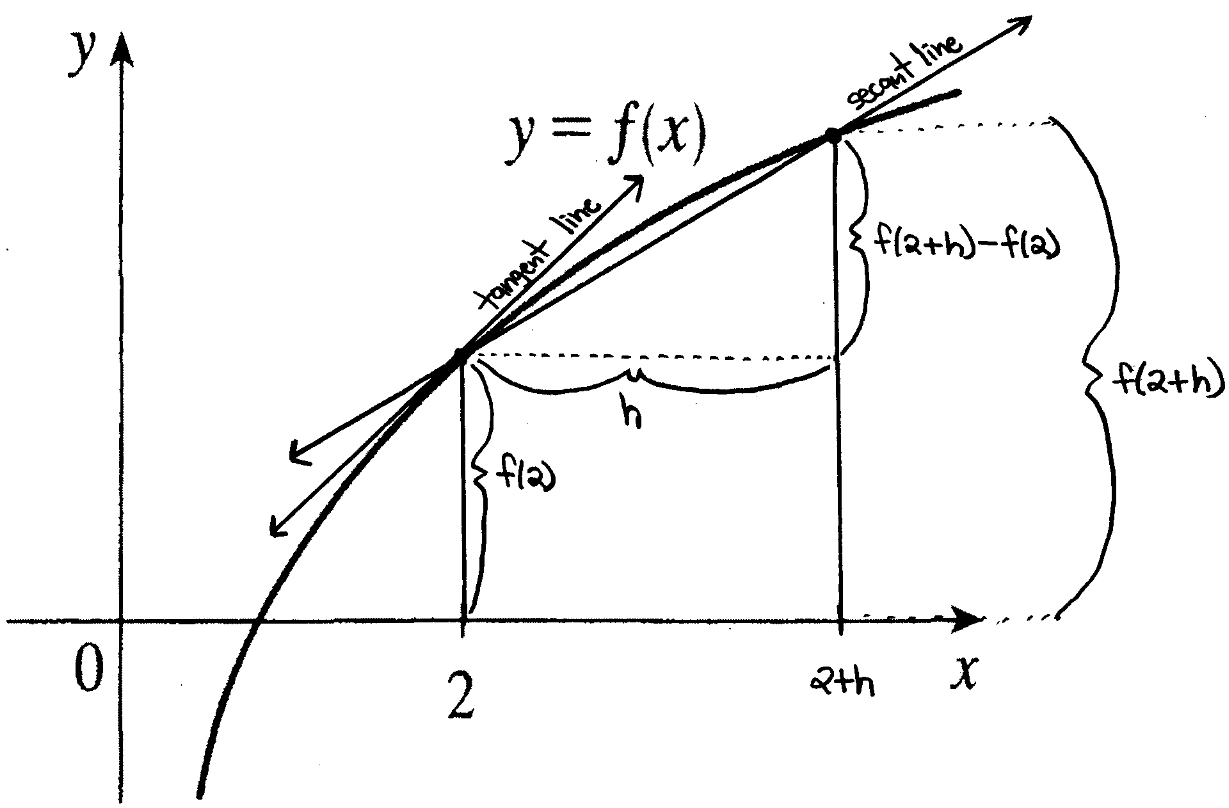


$\lim_{x\to 2}[f(x)+g(x)] =$ 2 $\quad \lim_{x\to 1}[f(x)+g(x)] =$ DNE $\quad \lim_{x\to 0}[f(x)g(x)] =$ 0

$\lim_{x\to -1}\dfrac{f(x)}{g(x)} =$ DNE $\quad \lim_{x\to 2} x^3 f(x) =$ 16 $\quad \lim_{x\to 1}\sqrt{3+f(x)} =$ 2

8. ~~9.~~ On the graph of f below, mark and label lengths that represent each of the following quantities. Choose h to be any quantity bigger than zero. (4 points)

- h
- $f(2)$
- $f(2+h)$
- $f(2+h)-f(2)$



Draw on your graph the line that has slope $\dfrac{f(2+h)-f(2)}{h}$. (2 points)

" slope of secant line

Draw on your graph the line that has slope $\lim_{h \to 0} \dfrac{f(2+h)-f(2)}{h}$. (2 points)

" slope of tangent line at $x=2$.

9. If $f(x) = x^3 - 9x^2 + 23x$ then we can use the Intermediate Value Theorem to show that there is a number c such that $f(c) = 15$. We will outline how this can be done below.

I. Find two distinct real numbers a and b such that $f(b) > 15$ and $f(a) < 15$ and $f(a) \neq f(b)$ (2 points)

Take $a=4$ so $f(a)=f(4)=12<15$

$b=6$ so $f(b)=f(6)=30>15$

II. We know that $f(x)$ is a polynomial function. What special property do ALL polynomial functions enjoy on ANY closed interval that relates to the IVT? (1 point)

Continuity

III. Use algebra or your graphing calculator to find a number c inside the interval (a, b) such that $f(c) = 15$. (2 points)

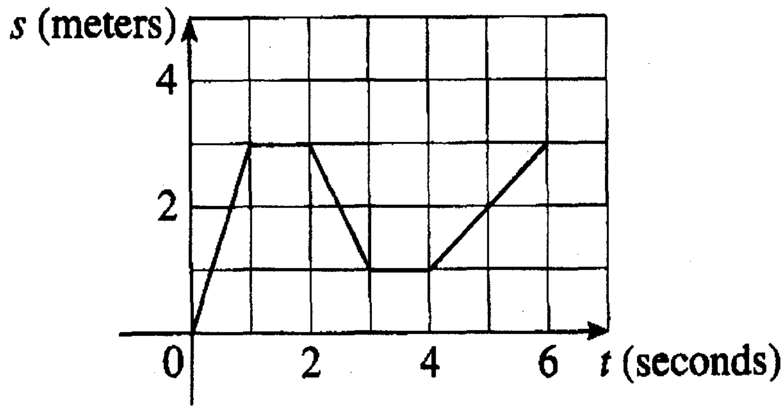
$c=5$ with $f(5)=15$

and 5 lies in the interval $(4,6)$.

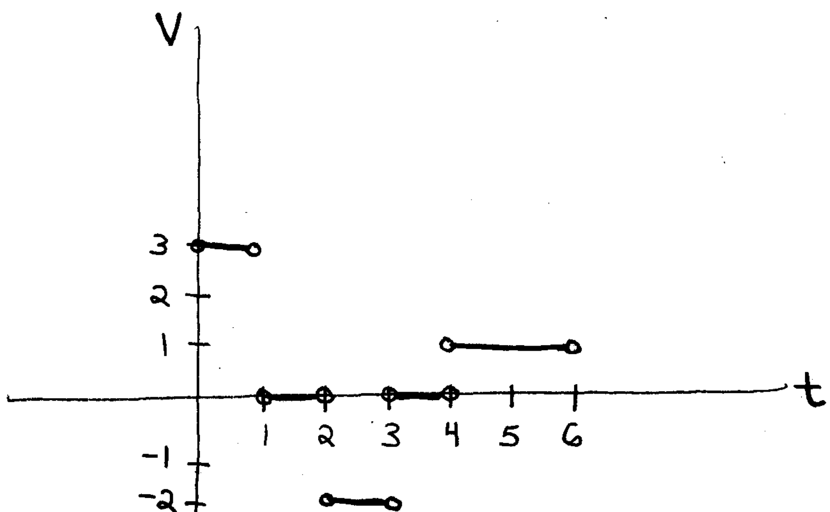
IV. List ALL numbers c that satisfy $f(c) = 15$ (1 point)

1, 3, and 5

10. A particle starts by moving to the right along a horizontal line; the graph of its position function is shown below.



Part a) Make a set of axes and sketch the graph of the velocity function. (4 points)



Part b) During what time interval is the particle moving to the left? (1 point)

From $t=2$ to $t=3$

Part c) During what time intervals is the particle at rest? (1 point)

From $t=1$ to $t=2$

and

From $t=3$ to $t=4$